

Imposing a 4-Dimensional Background on General Relativity

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Abstract. It is postulated that there exists a flat background spacetime whose increments can be directly measured by an observer, and on which local curved spacetimes exist. To make this premise work it must also be postulated that large spatial distances are measured by an observer using parallax and that the solutions to the Einstein Field Equations may be distorted to have a flat background. The mathematics of deriving background metrics and doing distortions are discussed. The distortion of the Schwarzschild Solution and some of its effects are discussed, including the Schwarzschild coordinates $r \leq 2m$ being made inaccessible.

INTRODUCTION

Although it is well accepted and verified, General Relativity (GR)[1, 2, 3] is not without its problems. Among them are the singularities of black holes[4], reachable positions at which the math of GR breaks down.

In this article, it is assumed that solving the Einstein Field Equations (EFE) is only part of the process of generating local metrics of spacetime, with the behavior of the black hole spacetimes indicating that this is not the final step in constructing metrics. To move beyond the EFE, it is also assumed that there exists a flat background spacetime to which the local spacetimes must conform, as described below. The resultant theory calls for the local metrics to be distortions of the EFE Solutions. The geodesic equations of the distorted metrics remove black holes by making their part of the manifold inaccessible.

POSTULATES AND THE OVERALL PREMISE

Postulate 1: There exists a background spacetime which is Minkowski in form, has intervals whose relative lengths are directly measured by an observer in the spacetime using their local clock and rod, and on which the local curved spacetimes exist.

Postulate 1 requires that it be possible to differentiate between locally measured intervals and those as measured by a single observer. For example, the temporal coordinate of the Schwarzschild Solution is a background time: It is based on the observations of a distant observer and its intervals can differ from those of the local clock. However, for spatial intervals there is a problem since the use of rods at rest[5, 6] forces all at-rest observers to agree on spatial lengths. This requires the next postulate:

Postulate 2: The spatial distance to a distant position is measured by the parallax of that position along a local baseline whose length is measured using the local rod.

The use of parallax to measure large distances is justified in [7]. Its use creates a gravitational length contraction effect, which complements gravitational time dilation and causes spatial distances to be increased for observers at lower potentials.

As shown later, the background spacetimes for curved EFE solutions usually are not flat (as required by Postulate 1). So it is also assumed that:

Postulate 3: EFE solutions correctly describe submanifolds of equal maximal clock rate but not the separations between those submanifolds.

Overall Premise: The local metrics of spacetime are distortions of the EFE solutions in the direction of changing maximal clock rate and/or parallaxically measured rod length such that their background spacetime is flat.

MATHEMATICAL REPRESENTATION

Extended Tensor Syntax

To support this discussion, an extended tensor syntax is used: $\text{source}^{\text{type}} X_{\text{form}}^{\text{vector indices}}$, where *type* is a code defining the type of values in a submetric of a metric tensor, and *source* is the identifying symbol of a metric tensor from which this tensor arises. (Example: ${}^t_g ds^2$ is the spacetime interval squared for the temporal submetric of the metric $g_{\mu\nu}$).

The background spacetime and its metric

From Postulate 1: “There exists a background spacetime ...”. This means that there exists a background spacetime interval B and a background metric $b_{\mu\nu}$ such that

$$dB^2 = b_{\mu\nu} dx^\mu dx^\nu, \quad (1)$$

where x^μ are the same coordinates used with the corresponding local metric or EFE Solution ($g_{\mu\nu}$).

Also from Postulate 1: The background spacetime “has intervals ... [that are] measured ... using [one’s] local clock and rod”. Given that at a distant position at a lower potential in a gravitational field there is

- a gravitational time dilation effect which decreases the observed temporal intervals between events,
- a gravitation length contraction effect (due to Postulate 2) which is of the same magnitude as the gravitational time dilation and increases the observed spatial distance between events[7], and
- a maximal relative rate at which a clock at the distant position may be observed to run (τ^*) which is also the magnitude of the gravitational time dilation and length contraction effects,

it follows that the background and local metrics are related by:

$$b_{\mu\nu} = {}^t g_{\mu\nu} / (\overset{*}{\tau})^2 + {}^s g_{\mu\nu} (\overset{*}{\tau})^2, \quad (2)$$

where ${}^t g_{\mu\nu}$ is the time-like submetric of the local metric ($g_{\mu\nu}$) and ${}^s g_{\mu\nu}$ is the space-like submetric of $g_{\mu\nu}$. How metrics are split into time-like and space-like submetrics is discussed in [8], and briefly in the example for the accelerated box below.

Distortion

From the Overall Premise: “The local metrics of spacetime are distortions of the EFE solutions ...”. Distortion is an operation that maintains the coordinates but changes the geometry. It is represented by a mixed rank-two tensor $N^\mu{}_\nu$ such that $h_{\mu\nu} = g_{\mu\alpha} N^\alpha{}_\nu$, where $g_{\mu\nu}$ is the original metric tensor and $h_{\mu\nu}$ is its distortion.

The distortion tensor is

$$N^\mu{}_\nu = \delta^\mu_\nu + S^\mu{}_\nu \quad (3)$$

where δ^μ_ν is the Kronecker delta and $S^\mu{}_\nu$ is the stretching tensor. $S^\mu{}_\nu$ in turn is

$$S^\mu{}_\nu = Q \hat{s}^\mu \otimes \hat{s}_\nu \quad (4)$$

where Q is the coefficient of stretching¹, \hat{s}^μ is a unit vector oriented in the direction of stretching, and \hat{s}_ν is a unit form oriented in the direction of stretching. Q , \hat{s}^μ , and \hat{s}_ν may be functions of position.

Implementation of the Overall Premise

The Overall Premise states that the distortion is “in the direction of maximal clock time ...”. For the stretching tensor definition given in (4), this indicates that

$$\hat{s}^\mu = \overset{*}{\tau}{}^{,\mu} / \left| \overset{*}{\tau}{}^{,\mu} \right| \quad \text{and} \quad \hat{s}_\nu = \overset{*}{\tau}_{,\nu} / \left| \overset{*}{\tau}_{,\nu} \right|, \quad (5)$$

where “,” is the covariant derivative operation.

The Overall Premise also states that the distortion is “such that the background becomes flat”. So given a distorted background metric $i_{\mu\nu}$ such that

$$i_{\mu\nu} = b_{\mu\alpha} N^\alpha{}_\nu = b_{\mu\alpha} (\delta^\alpha_\nu + Q \hat{s}^\alpha \otimes \hat{s}_\nu), \quad (6)$$

there exists at each position in the spacetime a value for Q such that

$$iR_{\nu\mu\sigma\sigma} = 0 \quad (7)$$

¹ Q is related to the factor of change in the incremental distance between coordinates in the direction of stretching (n) by $Q = n^2 - 1$.

where ${}^i R_{\nu\mu\sigma\alpha}$ is the Riemann tensor for $i_{\mu\nu}$. In this article, only stationary spacetimes are discussed. In these cases, the direction of changing maximal clock time and therefore the distortion itself cannot have a time-like component. With only the space-like submetric being affected, a distortion of a local metric or EFE Solution will distort the background in the same way. Therefore the local metrics for stationary spacetimes are

$$j_{\mu\nu} = g_{\mu\alpha} N^\alpha{}_\nu \quad (8)$$

where $N^\alpha{}_\nu$ is the distortion used to flatten the background metric for the EFE Solution $g_{\mu\alpha}$, obtained by using (2), (5), and (6), and then solving for Q in (7).

DISTORTIONS OF EFE SOLUTIONS

The Accelerated Box

The tetrad spacetime interval for an observer in Minkowski spacetime who is being accelerated in the $+z$ direction at a rate of a is[9]:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = (1 + az)^2 dt^2 - dx^2 - dy^2 - dz^2. \quad (9)$$

In (9), $\tau^* = 1 + az$, and this occurs when a particle is undergoing an incremental displacement of $d\xi^\mu = (dt, 0, 0, 0)$ [where the tensor indices are (t, x, y, z)]. As shown in [8], the temporal component of (9) is then

$${}^t ds^2 = g_{\mu\nu} d\xi^\mu d\xi^\nu = (1 + az)^2 dt^2, \quad (10)$$

and the spatial component of (9) is

$${}^s ds^2 = ds^2 - {}^t ds^2 = -dx^2 - dy^2 - dz^2. \quad (11)$$

Using (1) and (2) with (10) and (11) gives a background spacetime interval for (9) of

$$dB^2 = dt^2 - (1 + az)^2(dx^2 + dy^2 + dz^2). \quad (12)$$

It turns out that the Riemann tensor for (12) is null (${}^b R_{\mu\nu\sigma\alpha} = 0$). This means that Minkowski spacetime is not affected, and that this theory maintains GR's correspondence with Special Relativity for spacetimes far from gravitational sources.

The Schwarzschild Solution

The Schwarzschild Solution for the external vacuum spacetime surrounding a spherically symmetric non-rotating massive object is[10, 11]:

$$g ds^2 = (1 - 2m/r) dt^2 - (1 - 2m/r)^{-1} dr^2 - r^2 d\theta^2 - (r \sin \theta)^2 d\phi^2. \quad (13)$$

For (13), $\tau^* = \sqrt{1 - 2m/r}$, $d\xi^\mu = (dt, 0, 0, 0)$, and the background spacetime interval is

$$dB^2 = dt^2 - dr^2 - r(r - 2m)[d\theta^2 + (\sin\theta)^2 d\phi^2]. \quad (14)$$

The Riemann tensor for (14) is not null², and so (13) needs to be distorted to have a flat background. Since the τ^* for (13) is a function only of r , the stretching tensor for (14) obtained using (4) and (5) is $S^r_r = -Q(r)$ with all other terms being zero³. (3) then gives a distortion tensor of

$$N^r_r = 1 - Q(r), \quad N^t_t = N^\theta_\theta = N^\phi_\phi = 1, \quad \text{all other terms } 0. \quad (15)$$

(6) and (15) produce a distorted background for (14) of

$$dB^2 = i_{\mu\nu} dx^\mu dx^\nu = dt^2 - [1 - Q(r)]dr^2 - r(r - 2m)[d\theta^2 + (\sin\theta)^2 d\phi^2]. \quad (16)$$

The independent non-zero terms of the Riemann tensor for (16) ($iR_{\mu\nu\sigma\phi}$) are

$$iR_{r\theta r\theta} = \frac{1}{2} \left[\frac{d}{dr} Q(r) \right] (r - m) / [1 - Q(r)] - m^2 / [r(r - 2m)], \quad (17)$$

$$iR_{r\phi r\phi} = iR_{r\theta r\theta} (\sin\theta)^2, \quad \text{and} \quad (18)$$

$$iR_{\theta\phi\theta\phi} = (\sin\theta)^2 [Q(r)r^2 - 2rQ(r)m + m^2] / [1 - Q(r)]. \quad (19)$$

(17), (18), and (19) all go to zero [as required by (7)]when

$$Q(r) = -m^2 / [r(r - 2m)]. \quad (20)$$

(8), (15), and (20) then produce a local metric ($j_{\mu\nu}$) for (13) whose spacetime interval is

$$_j ds^2 = (1 - 2m/r) dt^2 - [(r - m)^2 / (r - 2m)^2] dr^2 - r^2 d\theta^2 - (r \sin\theta)^2 d\phi^2. \quad (21)$$

(21) is referred to as the Distorted Schwarzschild Solution.

The Equations of Motion for the Distorted Schwarzschild Solution

Given that the metrics generated by (8) [such as (21)] are the true local metrics of spacetime, it follows that the geodesic principle applies with respect to them. Therefore

$$\ddot{x}^\mu + {}_j \Gamma^\mu_{\rho\sigma} \dot{x}^\rho \dot{x}^\sigma = 0 \quad (22)$$

where $\dot{x}^\mu = dx^\mu / d\tau$, $\ddot{x}^\mu = d^2 x^\mu / d\tau^2$, and ${}_j \Gamma^\mu_{\rho\sigma}$ are the connections for $j_{\mu\nu}$.

² as shown by (17), (18), and (19) when $Q(r) = 0$

³ The negative sign of S^r_r is due to the Lorentz signage being used.

The geodesic equations for (21) are

$$\ddot{t} + 2m\dot{t}/[r(r-2m)] = 0, \quad (23)$$

$$\begin{aligned} \ddot{r} + (r-2m)^2 m(\dot{t})^2 / [(r-m)^2 r^2] - m(\dot{r})^2 / [(r-2m)(r-m)] \\ - (r-2m)^2 r [(\dot{\theta})^2 + (\sin\theta)^2 (\dot{\phi})^2] / (r-m)^2 = 0, \end{aligned} \quad (24)$$

$$\ddot{\theta} + 2\dot{r}\dot{\theta}/r - \sin\theta \cos\theta (\dot{\phi})^2 = 0, \quad \text{and} \quad (25)$$

$$\ddot{\phi} + 2\dot{r}\dot{\phi}/r + 2\dot{\theta}\dot{\phi} \cos\theta / \sin\theta = 0. \quad (26)$$

For an object at spatial rest in the coordinate system, $\dot{r} = \dot{\theta} = \dot{\phi} = 0$, and $\dot{t} = \sqrt{r/(r-2m)}$. Substituting these values into (24) produces

$$\ddot{r} = -m(r-2m) / [r(r-m)^2]. \quad (27)$$

In (27), $\lim_{r \rightarrow 2m} \ddot{r} = 0$. So at $r = 2m$ there is no downward coordinate acceleration from spatial rest. Additionally, if $\dot{r} \neq 0$ then $\lim_{r \rightarrow 2m} \ddot{r} = \infty$. Due to these effects, the coordinate space of $r < 2m$ is inaccessible in the geometry of (21), and $2m$ is now the minimum permissible size of an uncharged object of geometrized mass m . Additional effects arising from the use of (21) instead of (13) are discussed in [12]. This includes a demonstration that in low gravitational fields the difference in the incremental distance between radial coordinates in (13) and (21) is $\frac{1}{2}m^2/r^2$.

CONCLUSION

In conclusion, a model is being presented here that prohibits the formation of black holes, and corresponds well to Einstein's GR in cases where this theory and Einstein's have been tested so far. More research is needed to verify its viability and find testable predictions that differ from those of Einstein's GR.

REFERENCES

1. Einstein, A., *Annalen der Physik*, **49** (1916), as translated in *The Principle of Relativity*[13] pp. 111–164.
2. Wald, R. M., *General Relativity*, The University of Chicago Press, Chicago 60637, 1984.
3. Ohanian, H. C., and Ruffini, R., *Gravitation and Spacetime*, W. W. Norton & Co., New York, NY, 1994, 2nd edn.
4. Wald, R. M., *General Relativity*, chap. 9, pp. 211–242, in [2] (1984).
5. Einstein, A., *Relativity*, Bonanza Books, New York, NY, 1961, chap. XXIV, pp. 83–86.
6. Schmidt, H.-J., How to measure spatial distances? (1995), available at <http://arXiv.org> as preprint gr-qc/9512006.
7. Schaefer, E. M., Why distance in relativity is measured using parallax (2002), available at <http://users.erols.com/ems57/Distortion/distance.html>.
8. Schaefer, E. M., On the splitting of spacetime metrics (2002), available at <http://users.erols.com/ems57/Distortion/splitting.html>.

9. Misner, C. W., Thorne, K. S., and Wheeler, J. A., *Gravitation*, W. H. Freeman and Co., San Francisco, CA, 1973, section 6.6, p. 173, eq. (6.18).
10. Wald, R. M., *General Relativity*, chap. 6, p. 124, in [2] (1984), eq. (6.1.44).
11. Ohanian, H. C., and Ruffini, R., *Gravitation and Spacetime*, chap. 7, pp. 393–396, in [3] (1994).
12. Schaefer, E. M., A study of the Distorted Schwarzschild Solution (2002), available at <http://users.erols.com/ems57/Distortion/distorted-Schwarzschild-study.html>.
13. Lorentz, H. A., Einstein, A., Minkowski, H., and Weyl, H., *The Principle of Relativity*, Dover Publications, Inc., 180 Varick Street, New York, NY 10014, 1952, translated by W. Perrett and G.B. Jeffery.