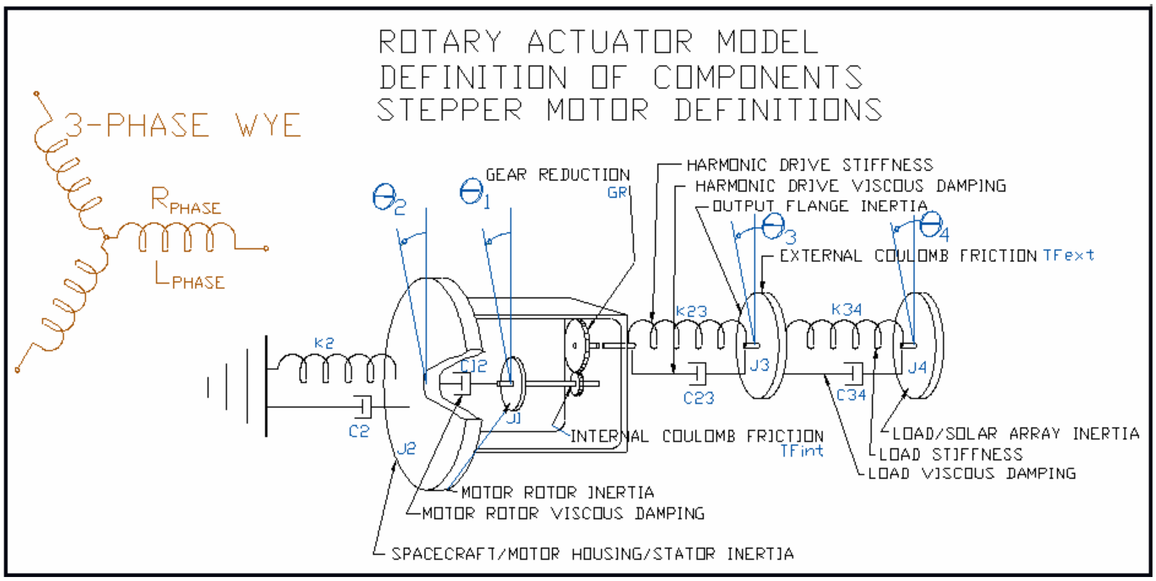
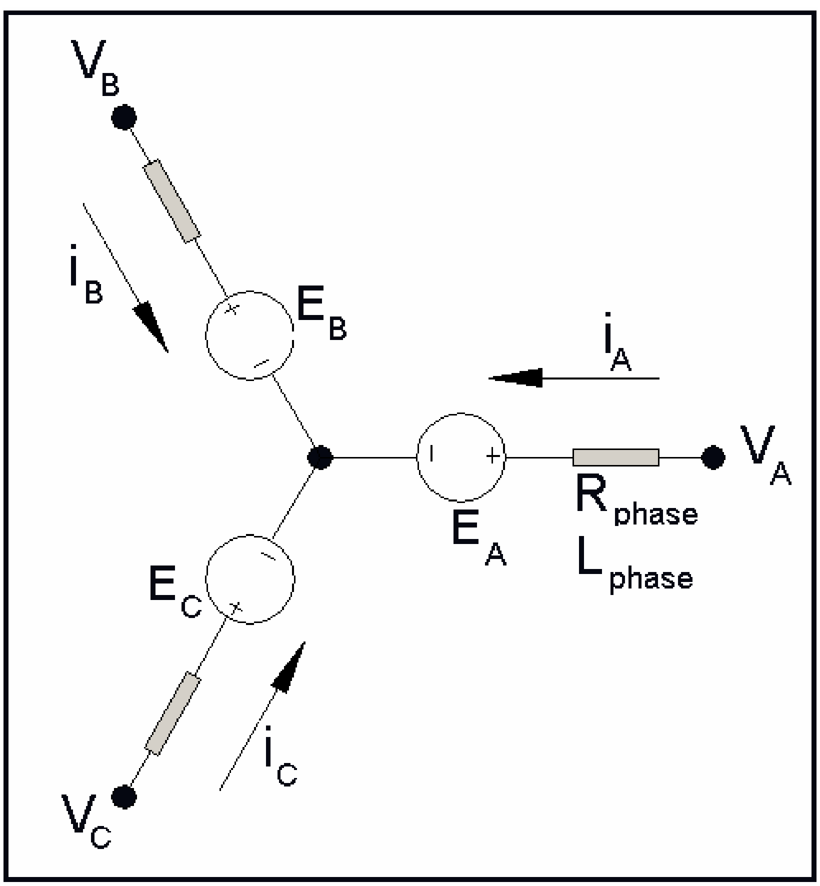
3-Phase Wye Stepper Motor Model

Rodger Farley 2008

The Model

This is a 4 degree of freedom electro-mechanical math model representing the motor rotor, the spacecraft, the output flange, and the flexible load (solar array).





3-phase-wye 6-state stepper motor

Applied voltages V_A, V_B, V_C.

Back emf voltages E_A, E_B, E_C

Phase current i_A, i_B, i_C

Phase resistance R_{phase}

Phase inductance L_{phase}

Phase Torque Constants

$Kt_{phase} := \frac{2}{3} \cdot K_T$ Maximum phase Torque constant from overall motor Kt, N-m/amp

1 leg to 2 leg standard operation

$$Kt_{instA}(\theta) := Kt_{phase} \cdot \sin\left(\frac{2\cdot\pi}{6} \cdot \frac{\theta}{Step_{size}}\right)$$

$$Kt_{instB}(\theta) := Kt_{phase} \cdot \sin\left(\frac{2\cdot\pi}{6} \cdot \frac{\theta}{Step_{size}} - \frac{4\cdot\pi}{3}\right)$$

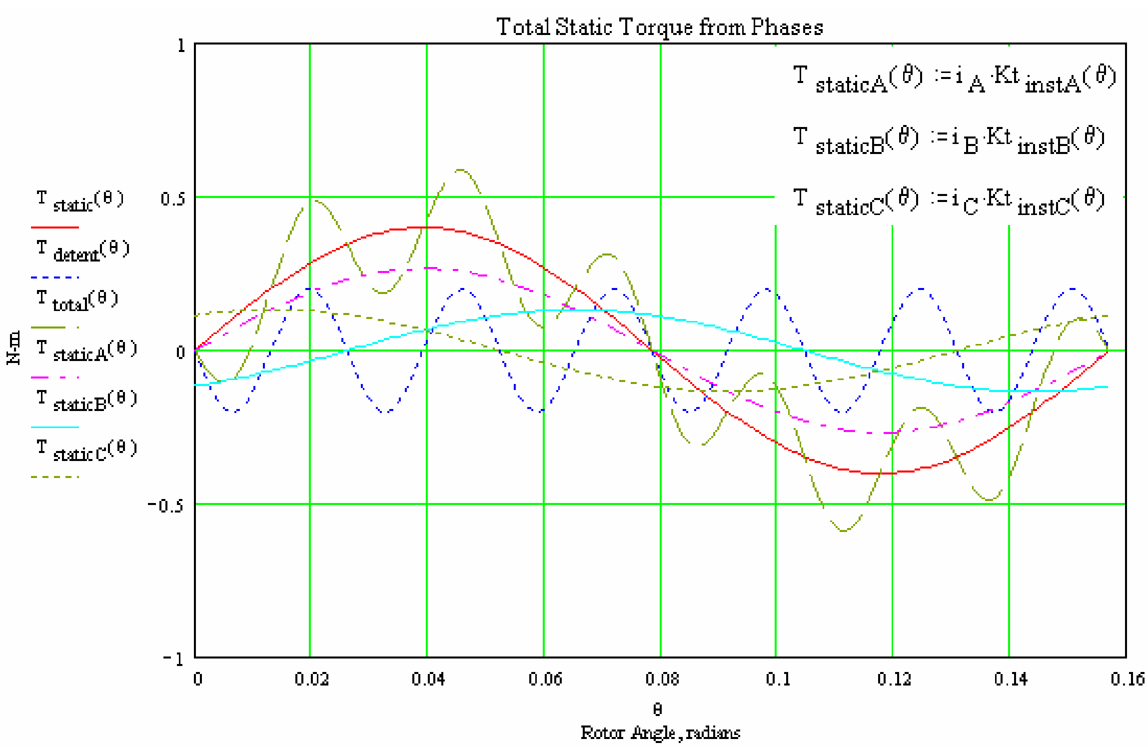
$$Kt_{instC}(\theta) := Kt_{phase} \cdot \sin\left(\frac{2\cdot\pi}{6} \cdot \frac{\theta}{Step_{size}} - \frac{2\cdot\pi}{3}\right)$$

Theta θ used here is the physical rotor angle, in the diagram = θ_1

The $Step_{size}$ for this motor = 1.5 degrees, or 0.026178 radians

Detent Torque

$$T_{detent}(\theta) := -Tdetent_{max} \cdot \sin\left(\frac{2\cdot\pi\cdot\theta}{Step_{size}}\right)$$

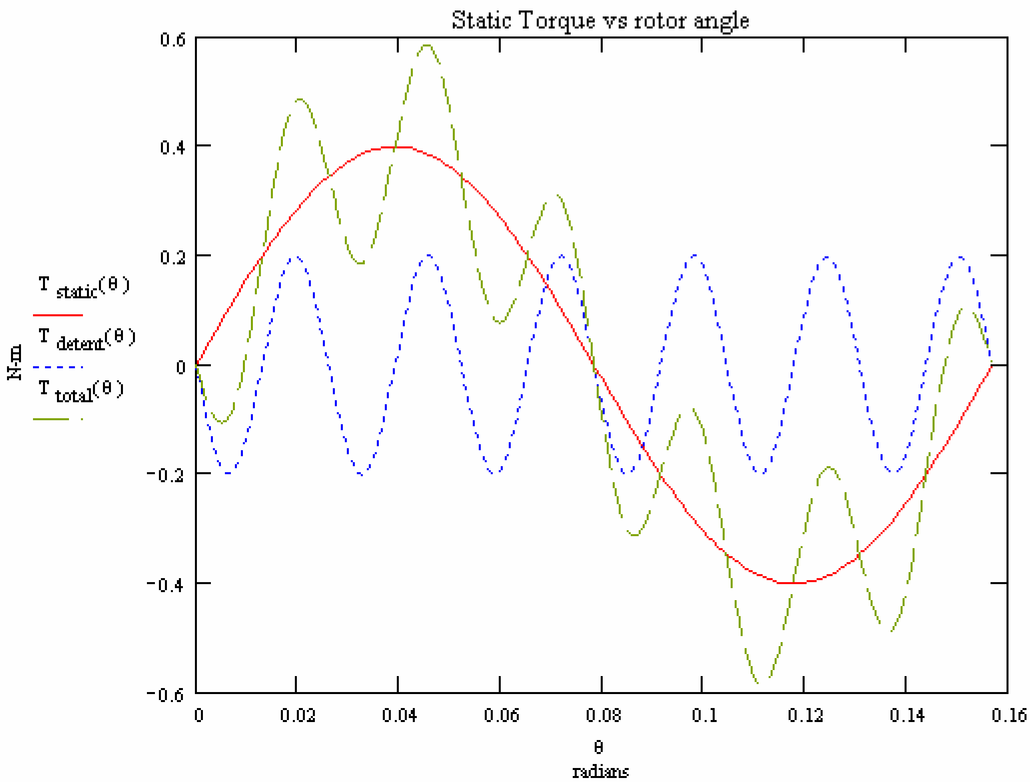


Static Torque Curve

With no transient effects and no back-emf effects, this is the shape of the static torque curve for $I_{circuit}$ = 0.2 amps:

$$T_{detent}(\theta) := -Tdetent_{max} \cdot \sin\left(\frac{2 \cdot \pi \cdot \theta}{Step_{size}}\right) \qquad T_{static}(\theta) := I_{circuit} \cdot K_T \cdot \sin\left(\frac{2 \cdot \pi \cdot \theta}{6 \cdot Step_{size}}\right)$$

$$T_{total}(\theta) := T_{static}(\theta) + T_{detent}(\theta)$$



Note the exaggerated effect of the detent when operating at such low currents.

Motor Stiffness

$$MotorStiffness(\theta) := \frac{d}{d\theta} T_{total}(\theta) := \left(\frac{\pi}{3 \cdot Step_{size}} \cdot I_{circuit} \cdot K_T \cdot \cos\left(\frac{1}{3} \cdot \pi \cdot \frac{\theta}{Step_{size}}\right)\right) - \left(\frac{2 \cdot \pi}{Step_{size}} \cdot Tdetent_{max} \cdot \cos\left(2 \cdot \pi \cdot \frac{\theta}{Step_{size}}\right)\right)$$

$$ReflectedStiffness(\theta) := MotorStiffness(\theta) \cdot GR^2$$

With 0.2 amps the reflected motor stiffness at the gear head output (but before the flexible portion of the harmonic drive) = 2.56x10^6 N-m/rad
Un-powered the reflected motor stiffness at the gear head output = 1.92x10^6 N-m/rad
This is extremely stiff (100x stiffer) when compared to the harmonic drive.

Back EMF voltage

A motor is also a generator, and generates a back voltage proportional to the speed of the rotor and the instantaneous value of the motor constant (which varies sinusoidally in a non-commutated motor such as a stepper motor).

$$E_A = -\frac{d\theta}{dt} \cdot Kt_{instA}$$

$$E_B = -\frac{d\theta}{dt} \cdot Kt_{instB}$$

$$E_C = -\frac{d\theta}{dt} \cdot Kt_{instC}$$

Transient Calculation of Phase Currents

The inductance creates transient current effects for the applied voltage:
The first 2 equations must be time-domain integrated.

$$\frac{di_A}{dt} = \frac{2 \cdot V_A - V_B - V_C}{3 \cdot L_{phase}} - \frac{R_{phase}}{L_{phase}} \cdot i_A + \frac{2 \cdot E_A - E_B - E_C}{3 \cdot L_{phase}}$$

$$\frac{di_B}{dt} = \frac{2 \cdot V_B - V_A - V_C}{3 \cdot L_{phase}} - \frac{R_{phase}}{L_{phase}} \cdot i_B + \frac{2 \cdot E_B - E_A - E_C}{3 \cdot L_{phase}}$$

$$i_C = -i_A - i_B$$

Net Motor Torque

$$T_{electrical} = i_A \cdot Kt_{instA} + i_B \cdot Kt_{instB} + i_C \cdot Kt_{instC}$$

$$T_{NET} = T_{electrical} + T_{detent}$$

Input Friction

Friction from the harmonic drive and motor bearings as a function of rotor speed and temperature, as seen by the motor rotor (N-m):

$$Tf_{int} = A_o + A_1 \cdot \left(\frac{d\theta_1}{dt} \right)^{A_2} \cdot 10^{A_3 \cdot (Temperature + A_4)}$$

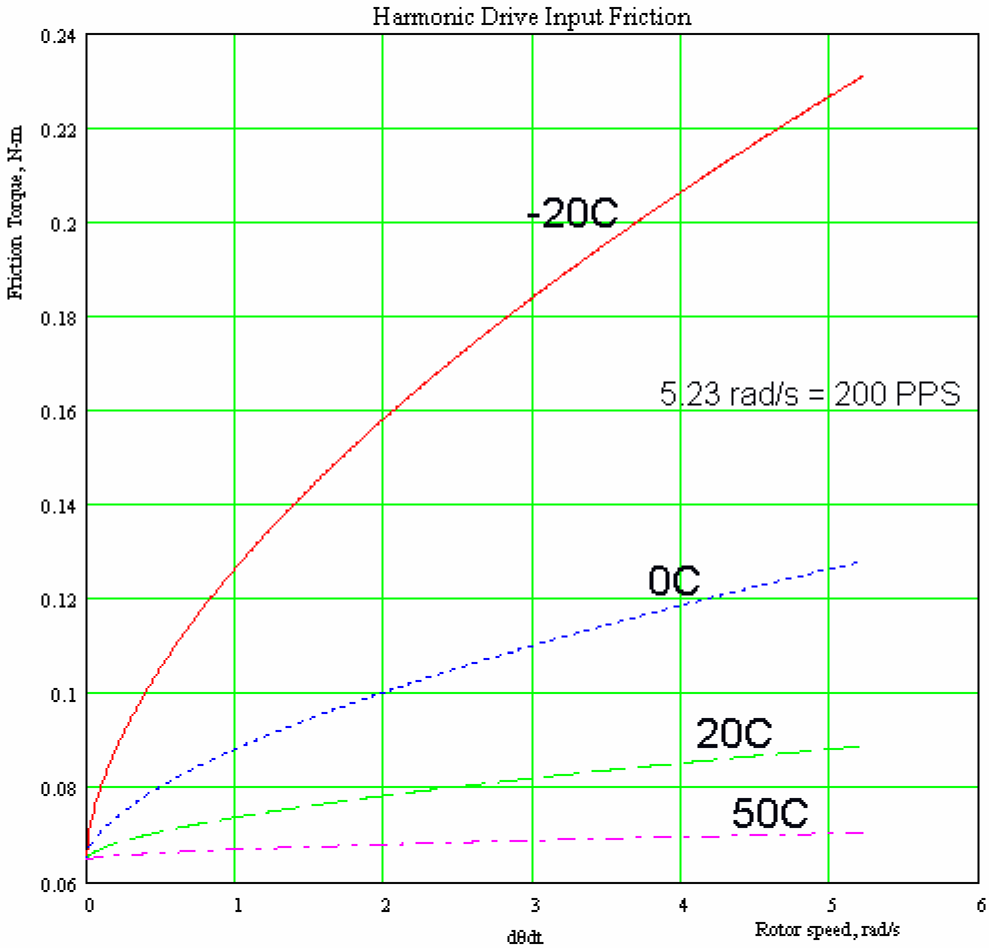
Ao is the static break-away torque at room temperature (21°C)

Type 5 size Harmonic Drive Friction Map, friction torque Qinput

$A_o := 0.065$ $A_1 := 0.0345$ $A_2 := 0.6$ $A_3 := -0.021$ $A_4 := 8.0$

dθdt is the rotor speed, radians per second

Temperature is the temperature in degC



Differential Equations of Motion

Note: Internal friction force Tfint is + when (dθ1/dt – dθ2/dt) is +
External friction force Tfext is + when (dθ3/dt – dθ2/dt) is +

Motor rotor

$$J_1 \cdot \frac{d^2\theta_1}{dt^2} = T_{NET} - Tf_{int}$$

$$-\frac{K_{23}}{GR} \cdot \left[\frac{\theta_1}{GR} - \theta_2 \cdot \left(1 + \frac{1}{GR}\right) + \theta_3 \right]$$

$$-\frac{C_{23}}{GR} \cdot \left[\frac{\frac{d\theta_1}{dt}}{GR} - \frac{d\theta_2}{dt} \cdot \left(1 + \frac{1}{GR}\right) + \frac{d\theta_3}{dt} \right] - C_{12} \cdot \left(\frac{d\theta_1}{dt} - \frac{d\theta_2}{dt} \right)$$

Spacecraft

$$J_2 \cdot \frac{d^2\theta_2}{dt^2} = -T_{NET} + Tf_{int} + Tf_{ext} + T_{environmental} + T_{ReactionWheels}$$

$$+K_{23} \cdot \left[\frac{\theta_1}{GR} - \theta_2 \cdot \left(1 + \frac{1}{GR}\right) + \theta_3 \right] \cdot \left(1 + \frac{1}{GR}\right)$$

$$+C_{23} \cdot \left[\frac{\frac{d\theta_1}{dt}}{GR} - \frac{d\theta_2}{dt} \cdot \left(1 + \frac{1}{GR}\right) + \frac{d\theta_3}{dt} \right] \cdot \left(1 + \frac{1}{GR}\right) + C_{12} \cdot \left(\frac{d\theta_1}{dt} - \frac{d\theta_2}{dt} \right)$$

Output Flange

$$J_3 \cdot \frac{d^2\theta_3}{dt^2} = -Tf_{ext}$$

$$-K_{23} \cdot \left[\frac{\theta_1}{GR} - \theta_2 \cdot \left(1 + \frac{1}{GR}\right) + \theta_3 \right] + K_{34} \cdot \left(\theta_4 - \theta_3\right)$$

$$-C_{23} \cdot \left[\frac{\frac{d\theta_1}{dt}}{GR} - \frac{d\theta_2}{dt} \cdot \left(1 + \frac{1}{GR}\right) + \frac{d\theta_3}{dt} \right] + C_{34} \cdot \left(\frac{d\theta_4}{dt} - \frac{d\theta_3}{dt} \right)$$

Flexible Load (solar array)

$$J_4 \cdot \frac{d^2\theta_4}{dt^2} = -K_{34} \cdot (\theta_4 - \theta_3) - C_{34} \cdot \left(\frac{d\theta_4}{dt} - \frac{d\theta_3}{dt} \right)$$

Stepper Motor States

The power is switched so as to provide voltage to the phases according to the following state table:

STATE	Phase A	Phase B	Phase C
1	+	+	-
2	+	-	-
3	+	-	+
4	-	-	+
5	-	+	+
6	-	+	-