

Free lift ratio FLratio = Gross Inflation / Gross weight = GI / G

Gross mass Mgross, kg

Ambient air density ρair, kg/m<sup>3</sup>

Air temperature to gas temperature ratio Tair/Tgas = Tairgas<sub>ratio</sub>

Shell thickness t<sub>shell</sub>, microns

Acceleration of gravity g = 9.807 m/s<sup>2</sup>

Drag coefficient Cdrag ~ 0.5

The air to gas temperature ratio Tairgas<sub>ratio</sub> is approximately ~ 1.05 below the trope and ~ 1.09 just before float.

In the case of ascent where the super pressure is zero, the free lift ratio can be expressed simply as:

$$FL_{ratio} = \frac{GI}{G} = \frac{M_{gas}}{M_{gross}} \cdot \left( \frac{1}{T_{airgas\_ratio}} \cdot \frac{R_{gas}}{R_{air}} - 1 \right)$$

Determine ambient density, volume, and balloon diameter at float:

$$\rho_{gas} = \rho_{air} \frac{R_{air}}{R_{gas}} \cdot T_{airgas\_ratio}$$

$$Volume = \frac{M_{gas}}{\rho_{gas}}$$

$$D := \left( \frac{6}{\pi} \cdot Volume \right)^{\frac{1}{3}} \quad \text{Spherical diameter, meters}$$

$$ROC := \frac{1}{D} \cdot \sqrt{\frac{8}{\pi} \cdot \frac{M_{gross} \cdot g \cdot (FL_{ratio} - 1)}{C_{drag} \cdot \rho_{air}}} \quad \text{Rate of Climb ROC, m/s}$$

The free lift may start at 10% but just before float it may be ~ 3.5% (FL<sub>ratio</sub> = 1.035)

### *Approximation Formulas for Air Density*

This is an approximation formula for air density as a function of height “h” in meters, good up to 23000m. It’s near perfect up to 11000m, and then overestimates up to 12% at 17000m. Best to use it in the range of 0 to 11000m.

$$AirDensity_{0to11km} = 1.225 \cdot \left[ 1 - \frac{h}{44303} \right]^{4.25}$$

This one follows very closely through out the range from 20 to 42 km:

$$AirDensity_{11to42km} = 2.077608 \cdot e^{-0.0001583 \cdot h}$$

Maximum error ~ +1.3% at 26000m and ~ -5% at 42000m