General Procedure for Designing Stratospheric Pumpkin Super Pressure Balloons

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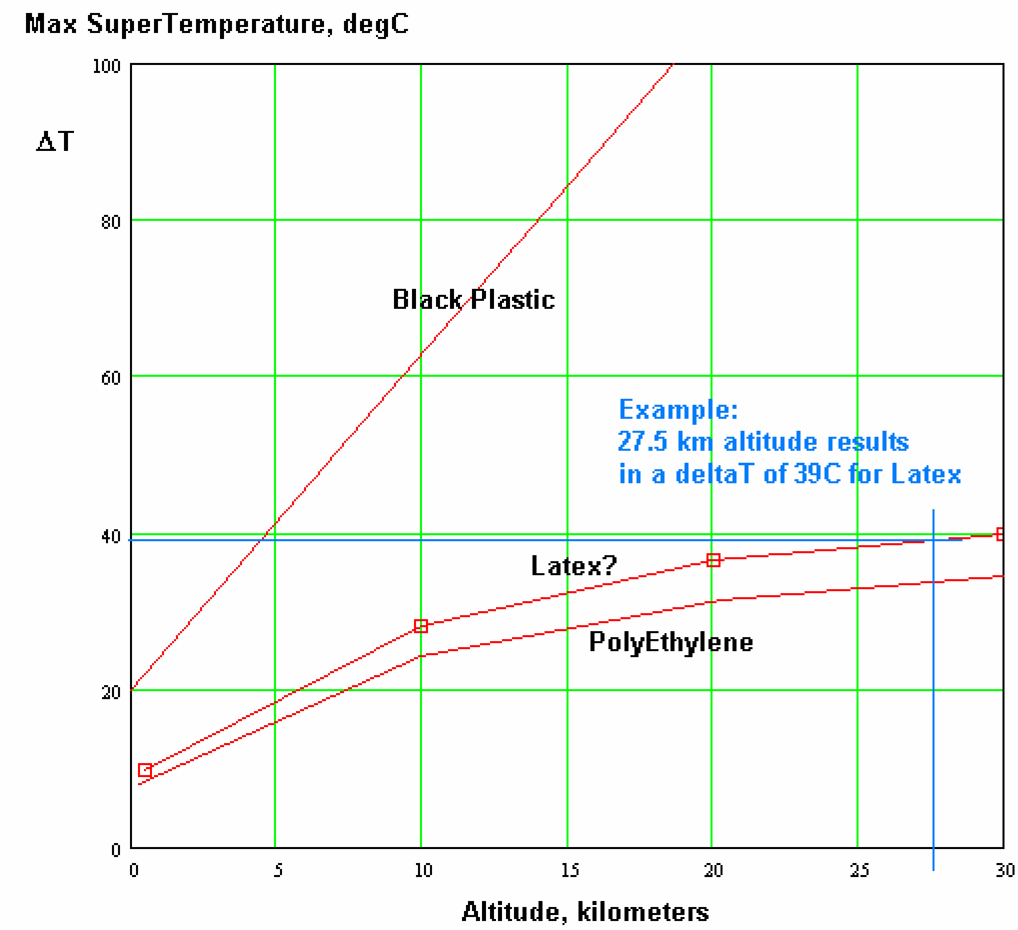
This paper describes a step-by-step process to design super pressure pumpkin balloons intended for flight in the stratosphere. A thermal analysis usually precedes the design phase in order to determine operating temperatures, but in lieu of that a general guide will be presented, good for the preliminary design stage.

Gas and film temperatures will be affected by the properties of the film; absorptivity, emissivity, and transmissivity; and by the environment. Absorptivity is a measure of a film's ability to absorb solar energy, emissivity is a measure of the films ability to radiate its own heat away in the form of infrared (IR) energy, and transmissivity is a measure of how much solar or IR energy simply passes right thru it. The environment that a balloon "sees" is divide into the direct solar, the reflected solar (albedo from surface and clouds), the IR coming up from the Earth's surface bathing the balloon from below, and the IR from a warm atmosphere bathing the balloon from above. There is also convection exchange between the balloon surface and the atmosphere. Some of the warmest environments is over the Antarctic due to the high albedo from the snow, almost giving the balloon a double sun (once from above and one more from below). The balloon can also experience very cold temperatures when it goes over high-topped cumulus clouds at night, the cloud acting as a blanket shielding the Earth's toasty IR from the balloon.

General Guide for Predicting Max Super Temperatures for Normal Conditions



We start with the basic givens

Preliminary Design Methodology of Super-Pressurized Pumpkin Balloons

Givens: $Volume := 270.1$ Desired pumpkin volume, cubic meters

$ALT_{float} := 29750$ Altitude at design float, meters

These gas temperature numbers are determined with a thermal analysis

$T_{day} := 264$ Average day time gas temperature, deg K (**design float case**)

$T_{night} := 224$ Average night time gas temperature, deg K

$R_{gas} := 4124$ Specific gas constant for the lifting gas, m^2/s^2/ deg K Hydrogen gas in this case

$R_{air} := 287.1$ Specific gas constant for the atmosphere, m^2/s^2/ deg K

$M_{apex} := 0.5$ Apex fitting mass, kg

$M_{nadir} := 0.5$ Nadir fitting mass, kg

$\rho_{film} := 920$ Film material mass density, kg/m^3

$m_{tendon} := 0.0005$ Tendon + sheath + seam linear density, kg/m

$thick_{shell} := 7$ Thickness of the shell film material, microns

$thick_{cap} := 0$ Thickness of the cap film material, microns

$LcapLgore := 0$ Cap length to gore length ratio

$g := 9.807$ Acceleration of gravity, m/s^2

$\alpha_B := \frac{95}{57.3}$ Desired bulge angle at equator, radians

$\sigma_{yield} := 8$ Yield stress of the film at the hot temperature, MPa

$FS := 1.5$ Yield factor of safety in the hoop direction for material stress

$\Delta P_{design} := 210$ Set the design pressure

Describe the atmosphere with a five-point model, 1st the temperature soundings

5-POINT ATMOSPHERE MODEL, TROPOSPHERE AND STRATOSPHERE

1962 Standard Atmosphere
deg K, meters, pascals

sea level temperature and pressure, pascals

$T_{SL} := 288.15$ $P_{SL} := 101325$

$T_1 := 215.65$ $H_1 := 11000$

$T_2 := 216.65$ $H_2 := 20000$

$T_3 := 228.65$ $H_3 := 32000$

$T_4 := 270.65$ $H_4 := 47000$

Atmospheric Lapse Rates, degK per meter

$L_1 := \left(\frac{T_1 - T_{SL}}{H_1}\right)$ Lapse rate #1 $L_1 = -6.591\cdot10^{-3}$

$L_2 := \left(\frac{T_2 - T_1}{H_2 - H_1}\right)$ Lapse rate #2 $L_2 = 1.111\cdot10^{-4}$

$L_3 := \left(\frac{T_3 - T_2}{H_3 - H_2}\right)$ Lapse rate #3 $L_3 = 1\cdot10^{-3}$

$L_4 := \left(\frac{T_4 - T_3}{H_4 - H_3}\right)$ Lapse rate #4 $L_4 = 2.8\cdot10^{-3}$

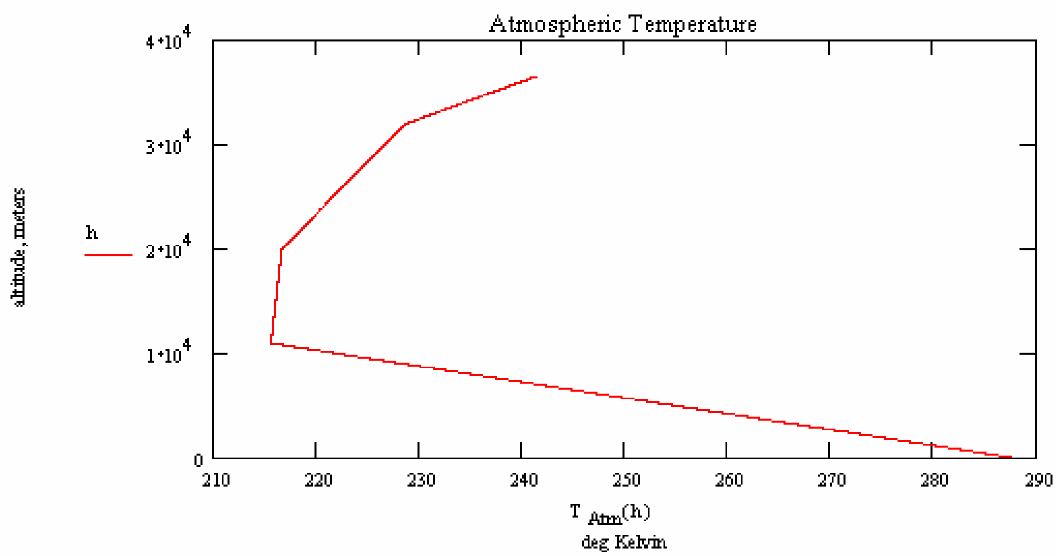
Standard AtmosphericTemperature, troposphere and stratosphere, deg K

$T_{1o}(h) := \text{if}\left[h < H_1, T_{SL} + L_1\cdot h, T_2 + L_2\cdot\left(h - H_2\right)\right]$ troposphere and near-isothermal stratosphere

$T_{2o}(h) := \text{if}\left[h < H_2, T_{1o}(h), T_3 + L_3\cdot\left(h - H_3\right)\right]$ lower stratosphere

$T_{Atm}(h) := \text{if}\left[h < H_3, T_{2o}(h), T_4 + L_4\cdot\left(h - H_4\right)\right]$ upper stratosphere

$h := 1, 10 .. 36585$



Next, the pressures are calculated:

Standard Atmospheric Pressure, troposphere and stratosphere, Pa

$$P_{1stpoint} := P_{SL} \cdot \left(\frac{T_1}{T_{SL}}\right)^{\frac{-g}{L_1 \cdot R_{air}}}$$

$P_{1stpoint} = 22562$ Pascals

$$P_1(h) := \mathrm{if}\left[h < H_1, P_{SL} \cdot \left(\frac{T_{Atm}(h)}{T_{SL}}\right)^{\frac{-g}{L_1 \cdot R_{air}}}, P_{1stpoint} \cdot \left(\frac{T_{Atm}(h)}{T_1}\right)^{\frac{-g}{L_2 \cdot R_{air}}}\right]$$

$$P_2(h) := \mathrm{if}\left[h < H_2, P_1(h), P_1\left(H_2\right) \cdot \left(\frac{T_{Atm}(h)}{T_2}\right)^{\frac{-g}{L_3 \cdot R_{air}}}\right]$$

$$P_{Atm}(h) := \mathrm{if}\left[h < H_3, P_2(h), P_2\left(H_3\right) \cdot \left(\frac{T_{Atm}(h)}{T_3}\right)^{\frac{-g}{L_4 \cdot R_{air}}}\right]$$

$$\rho_{Atm}(h) := \frac{P_{Atm}(h)}{R_{air} \cdot T_{Atm}(h)}$$

Air density in the troposphere and stratosphere, kg/m^3

$$\rho_{liftgas}(h, \Delta T, \Delta P) := \frac{P_{Atm}(h) + \Delta P}{R_{gas} \cdot \left(T_{Atm}(h) + \Delta T\right)}$$

Gas density as a function of ambient and differential temperature and pressure, kg/m^3

$$\rho_{LiftGas}\left(h, T_{gas}, \Delta P\right) := \frac{P_{Atm}(h) + \Delta P}{R_{gas} \cdot T_{gas}}$$

Gas density as a function of gas temperature, ambient and differential pressure, kg/m^3

Super pressure requirement

$\rho_{air} := \rho_{Atm}(ALT_{float})$	$\rho_{air} = 0.018609$	Air density at altitude, kg/m^3
$\rho_{gas} := \rho_{LiftGas}(ALT_{float}, T_{day}, \Delta P_{design})$	$\rho_{gas} = 1.304 \cdot 10^{-3}$	Helium density at altitude, kg/m^3
$T_{air} := T_{Atm}(ALT_{float})$	$T_{air} = 226.4$	Ambient temperature, K
$P_{air} := P_{Atm}(ALT_{float})$	$P_{air} = 1210$	Ambient pressure, Pa
$\Delta T_{day} := T_{day} - T_{air}$	$\Delta T_{day} = 37.6$	Daytime super temperature
$\Delta T_{night} := T_{night} - T_{air}$	$\Delta T_{night} = -2.4$	Nightime super temperature
$dPdT := \rho_{gas} \cdot R_{gas}$	$dPdT = 5.377$	For a constant density system, the rate of change of pressure with temperature Pascals / deg K
$dP_{daynight} := dPdT \cdot (T_{day} - T_{night})$	$dP_{daynight} = 215.1$	Normal pressure variation from night to day, Pa

The design super pressure should be at least this value

This particular example calculation has a recommendation of 215 Pa for the design super pressure.

Geometric and load values

$$R_{spherical} := \left(\frac{0.75\cdot Volume}{\pi}\right)^{0.333333}$$

$R_{spherical} = 4.01$ Equivalent spherical balloon radius, m

$$\frac{\sigma_{yield}}{FS} = 5.333$$

$$R_{bulge} := \frac{thick_{shell}}{\Delta P_{design}}\cdot\frac{\sigma_{yield}}{FS}$$

$R_{bulge} = 0.178$ Bulge radius to meet hoop stress requirement, meters

$$R_{balloon} := 1.146\cdot R_{spherical}$$

$R_{balloon} = 4.595$ Pumpkin balloon radius at the equator, m

The function "floor" means "make an integer of this number"

$$N_{gores} := floor\left[\frac{\pi}{asin\left(\frac{R_{bulge}}{R_{balloon}}\cdot sin\left(\frac{\alpha_B}{2}\right)\right)}\right]$$

$N_{gores} = 110$

Minimum number of gores required

$$GoreWidth_{max} := \alpha_B\cdot R_{bulge}$$

$GoreWidth_{max} = 0.295$ Max gore width, meters

$$TendonLoad := \frac{\pi\cdot R_{balloon}^{2}\cdot\Delta P_{design}}{N_{gores}}$$

$TendonLoad = 126.7$ Max tendon load, Newtons

Estimate the mass of the balloon envelope components

Pumpkin Balloon Mass Estimate of Component Parts

$Diameter_{balloon} := 2.257 \cdot R_{spherical}$ — $Diameter_{balloon} = 9.05$ — Approximate pumpkin diameter, m

$Height_{balloon} := 1.3875 \cdot R_{spherical}$ — $Height_{balloon} = 5.56$ — Approximate pumpkin height, m

$Gore_{length} := 2.96 \cdot R_{spherical}$ — $Gore_{length} = 11.87$ — Approximate theoretical gore length (including fitting radii) in the stressed state, m

$m_{shell} := \rho_{film} \cdot thick_{shell} \cdot 10^{-6}$ — Shell material area density, kg/m^2 — $m_{shell} = 6.44 \cdot 10^{-3}$

$m_{cap} := \rho_{film} \cdot thick_{cap} \cdot 10^{-6}$ — Cap material area density, kg/m^2 — $m_{cap} = 0$

$M_{shell} := m_{shell} \cdot 5.32 \cdot Volume^{\frac{2}{3}}$ — $M_{shell} = 1.4$ kg

$M_{cap} := m_{cap} \cdot 1.22 \cdot Diameter_{balloon}^{2} \cdot (1 - \cos(\pi \cdot LcapLgore))$ — $M_{cap} = 0$ kg

$M_{tendon} := m_{tendon} \cdot Gore_{length} \cdot N_{gores}$ — $M_{tendon} = 0.7$ kg

$M_{reefingsleeve} := 0.081 \cdot Diameter_{balloon}$ — $M_{reefingsleeve} = 0.7$ kg

$M_{wires} := 0.222 \cdot Gore_{length}$ — $M_{wires} = 2.6$ kg

$M_{InflationTubes} := 0.04 \cdot Gore_{length}$ — $M_{InflationTubes} = 0.5$ kg

$M_{destruct} := 0.0124 \cdot Gore_{length}$ — $M_{destruct} = 0.1$ kg

$M_{extra} := M_{wires} + M_{InflationTubes} + M_{destruct} + M_{reefingsleeve}$ — $M_{extra} = 3.99$ kg

For this case, Mextra = 0 — $M_{extra} := 0$

$M_{balloon} := M_{shell} + M_{cap} + M_{tendon} + M_{apex} + M_{nadir} + M_{extra}$

$M_{balloon} = 3.1$ — Approximate balloon material mass, kg

Small pumpkins will not have reefing sleeves, or destruct, or wires, or inflation tubes.

Calculate overall performance values

Initial Performance Parameters

$b := g \cdot (\rho_{air} - \rho_{gas})$ — $b = 0.169713$ — Specific buoyancy at design float conditions, N/m^3

$GrossLift := b \cdot Volume$ — $GrossLift = 45.8$ — Gross lift at design float conditions, N

$M_{gross} := \frac{GrossLift}{g}$ — $M_{gross} = 4.674$ — Gross mass of the balloon system (not including gas mass) for equilibrium flight at the design float conditions, kg

$M_{gas} := \rho_{gas} \cdot Volume$ — $M_{gas} = 0.352$ — Mass of lifting gas, kg

$M_{suspend} := M_{gross} - M_{balloon}$ — $M_{suspend} = 1.59$ — Suspended mass capacity at design float, kg

$FL_{SL} := 1.1$ — Minimum desired free ratio lift at launch, 10% typical (1.10)

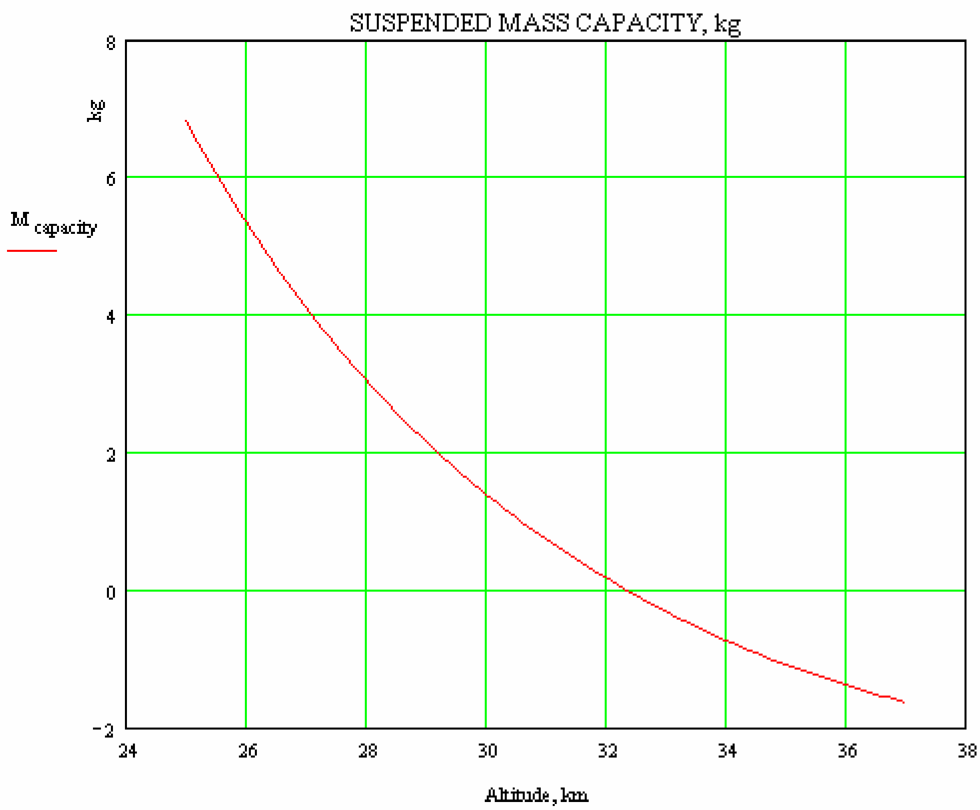
$M_{gasMIN} := M_{gross} \cdot \frac{FL_{SL}}{\left(\frac{R_{gas}}{R_{air}} - 1\right)}$ — $M_{gasMIN} = 0.385$ — Minimum mass of gas at launch to achieve the desired launch free lift ratio, kg

Graph of the suspended mass capacity as it varies with float altitude

SUSPENDED MASS CAPACITY AT DIFFERENT ALTITUDES
BASED ON THE NORMAL DAYTIME FLOAT TEMPERATURE
AND SUPERPRESSURE, CUSTOMIZED FOR EACH ALTITUDE

$\rho_{Gas}(h) := \rho_{LiftGas}(h, T_{day}, \Delta P_{design})$ Gas density if the temperature and super pressure are constant

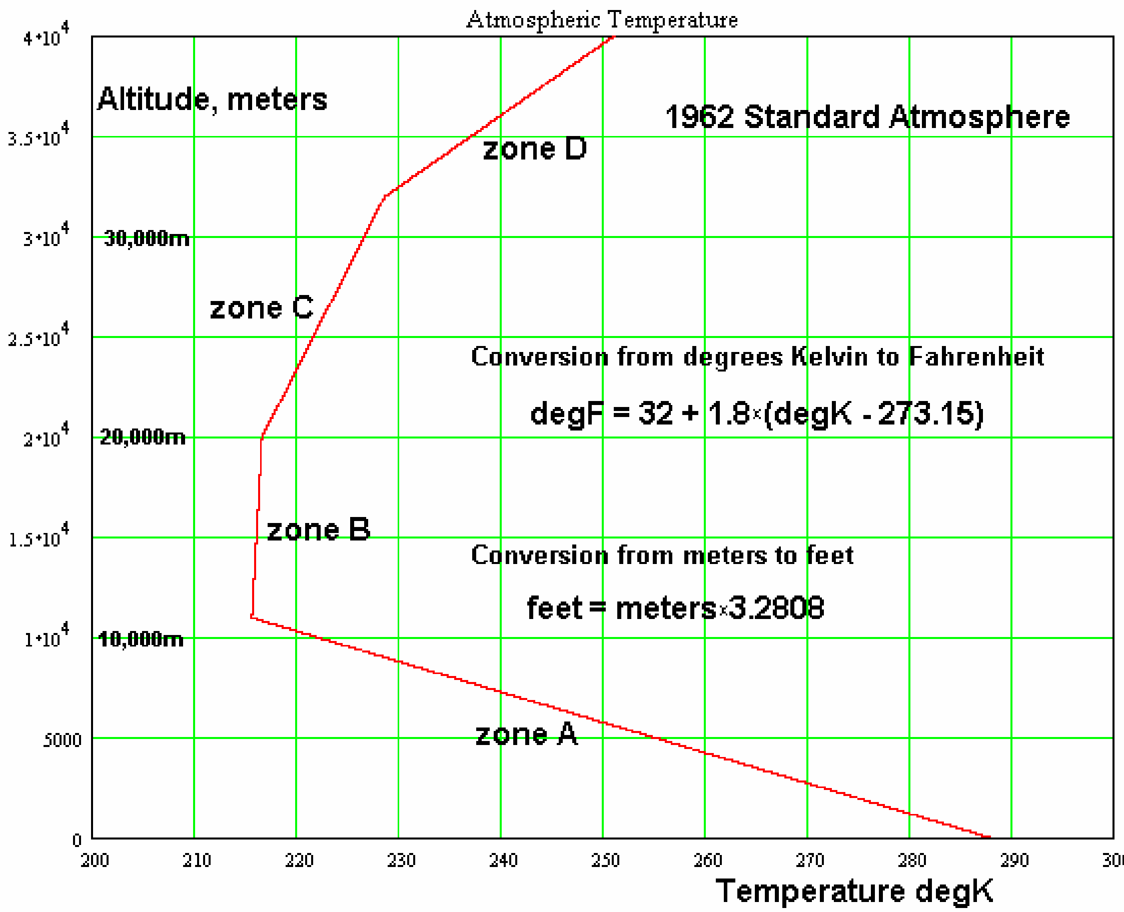
$M_{capacity}(h) := Volume \cdot (\rho_{Atm}(h) - \rho_{Gas}(h)) - M_{balloon}$



$M_{capacity}(36585) = -1.52$	kg	Suspended mass at 120000 ft
$M_{capacity}(33528) = -0.528$	kg	Suspended mass at 110000 ft
$M_{capacity}(30480) = 1.083$	kg	Suspended mass at 100000 ft
$M_{capacity}(27432) = 3.655$	kg	Suspended mass at 90000 ft

Appendix 1
Equations and Graphs for the Standard Atmosphere

Temperature



For zone A, altitude "h" (in meters) ranges from sea level to 11000m:

$$Temperature = 288.15 - 0.006591 \cdot h \text{ , in degrees K}$$

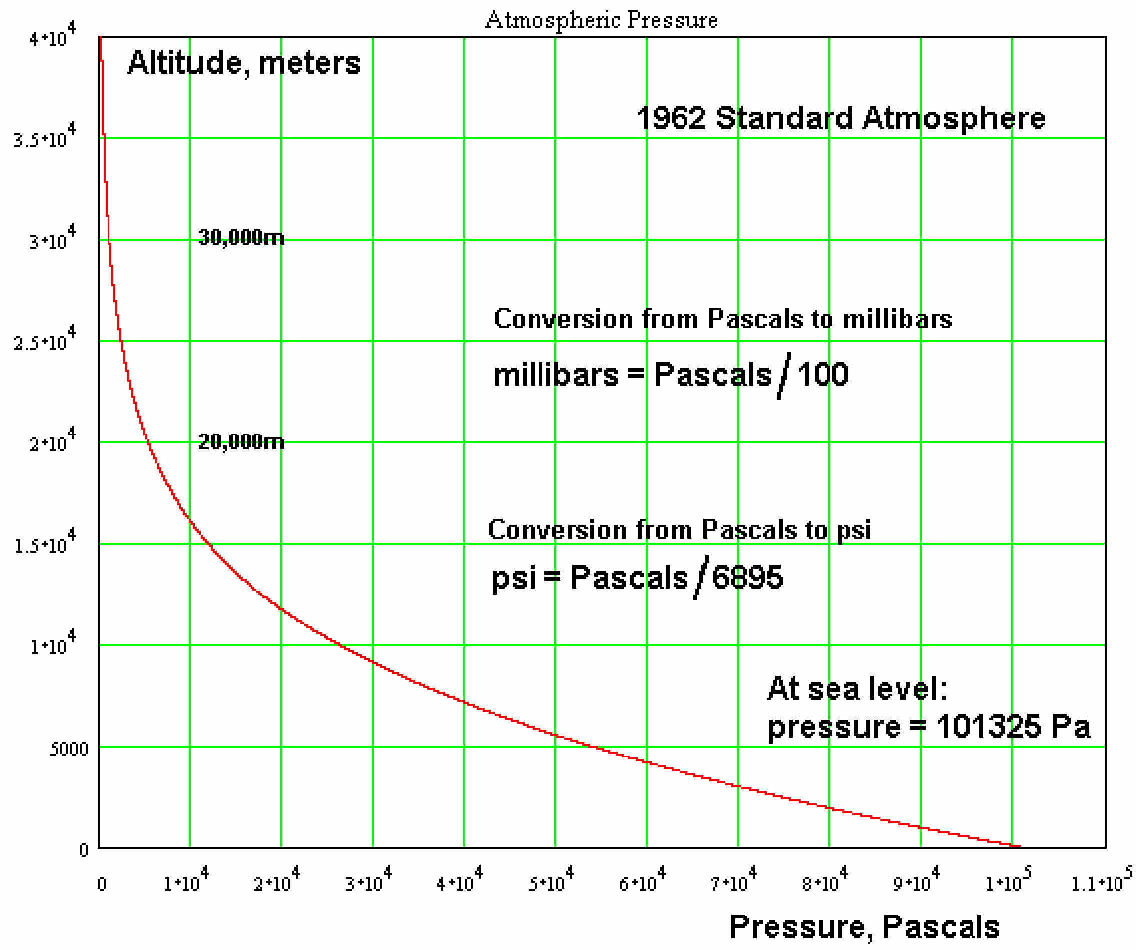
For zone B, altitude "h" ranges from 11000 to 20000m:

$$Temperature = 216.65 + 0.0001111 \cdot (h - 20000) \text{, in degrees K}$$

For zone C, altitude "h" ranges from 20000 to 32000m:

$$Temperature = 228.65 + 0.001 \cdot (h - 32000) \text{, in degrees K}$$

Pressure



For zone A, altitude "h" ranges from sea level to 11000m:

$$Pressure = 101325 \cdot \left[\frac{Temperature}{288.15}\right]^{5.1826} \quad \text{in Pascals}$$

For zone B, altitude "h" ranges from 11000 to 20000m:

$$Pressure = 22562 \cdot \left[\frac{Temperature}{215.65}\right]^{-307.43} \quad \text{in Pascals}$$

For zone C, altitude "h" ranges from 20000 to 32000m:

$$Pressure = 5441 \cdot \left[\frac{Temperature}{216.65}\right]^{-34.159} \quad \text{in Pascals}$$

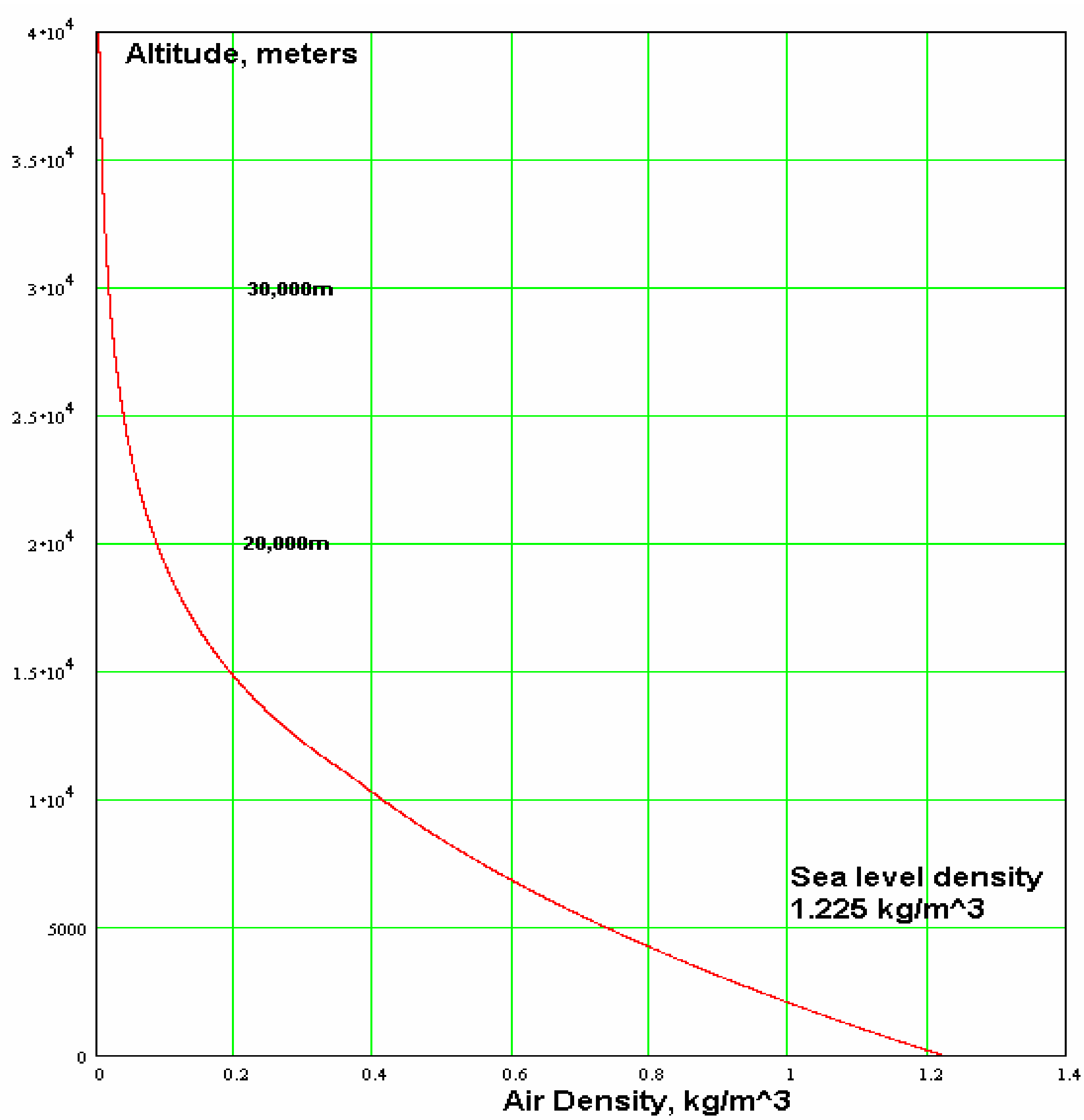
Use the appropriate temperature in deg K from calculated from the proper zone

Air Density

The air density is calculated with the ideal gas law:

$$AirDensity = \frac{Pressure}{287.1 \cdot Temperature} \quad \text{kilograms per cubic meter}$$

Where Pressure is in Pascals and Temperature is in degrees Kelvin



Approximation Formulas for Air Density

This is an approximation formula for air density as a function of height "h" in meters, good up to 23000m. It's near perfect up to 11000m, and then overestimates up to 12% at 17000m. Best to use it in the range of 0 to 11000m.

$$AirDensity_{0to11km} = 1.225 \cdot \left[1 - \frac{h}{44303}\right]^{4.25}$$

This one follows very closely through out the range from 20 to 42 km:

$$AirDensity_{11to42km} = 2.077608 \cdot e^{-0.0001583 \cdot h}$$

Maximum error ~ +1.3% at 26000m and ~ -5% at 42000m

Density in kilograms per cubic meter