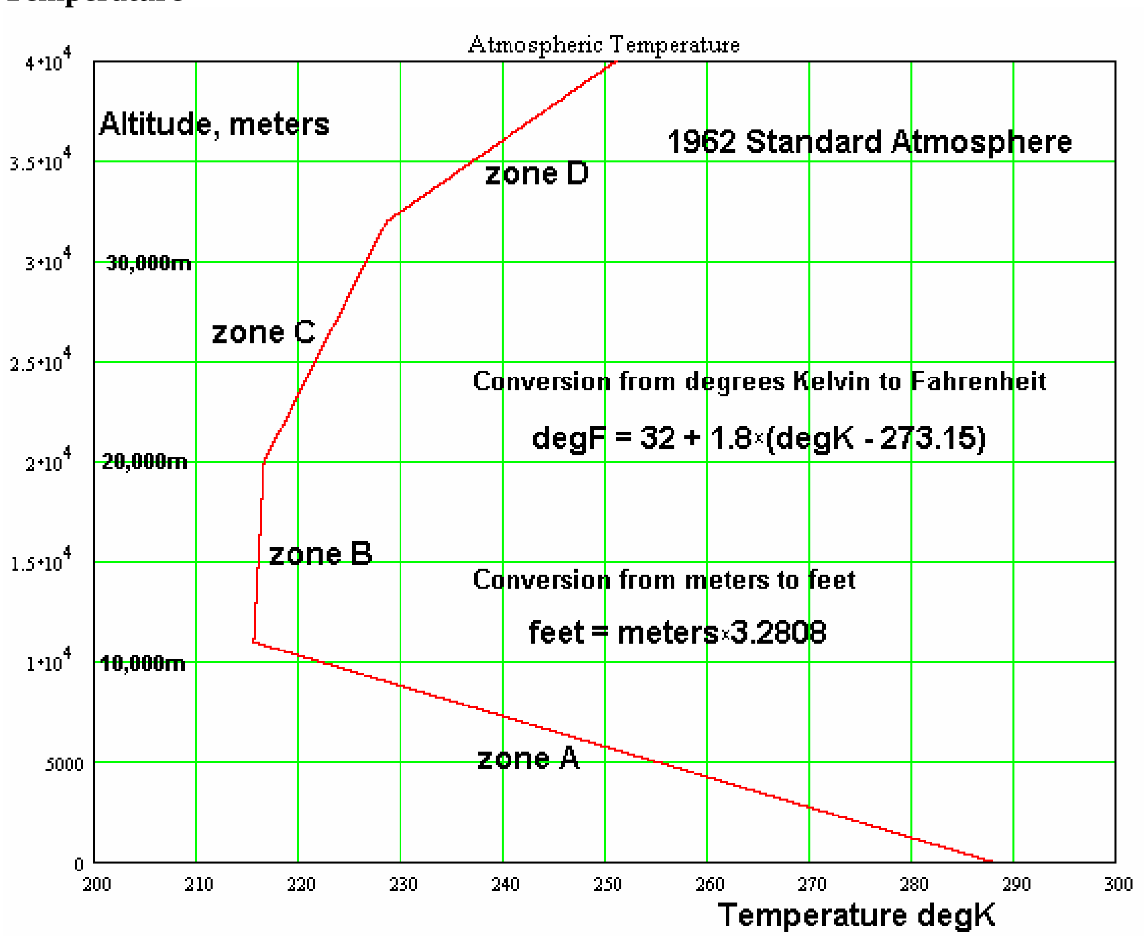
Equations and Graphs for the Standard Atmosphere and Equations for Solar Montgolfiere Balloons

Rodger Farley 2007

Temperature



For zone A, altitude “h” (in meters) ranges from sea level to 11000m:

$$Temperature = 288.15 - 0.006591 \cdot h \text{ , in degrees K}$$

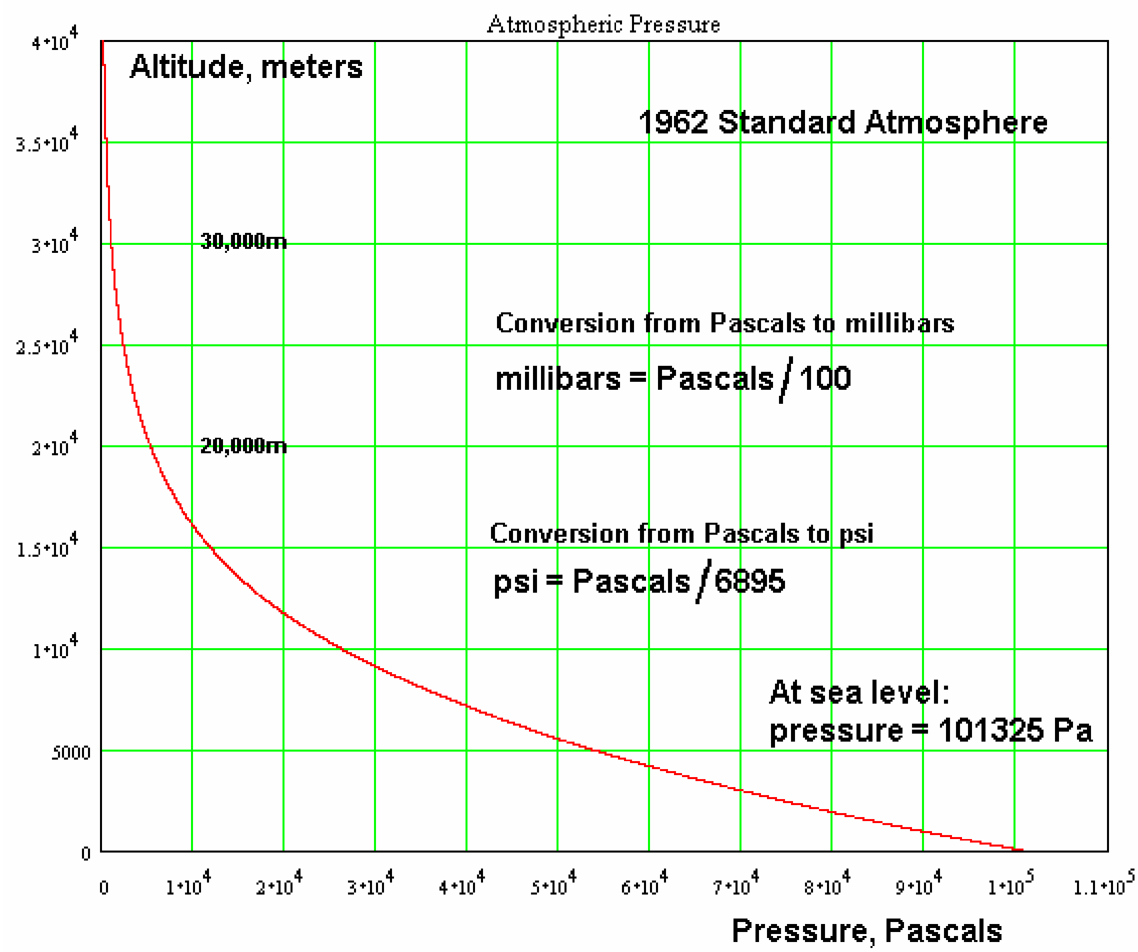
For zone B, altitude “h” ranges from 11000 to 20000m:

$$Temperature = 216.65 + 0.0001111 \cdot (h - 20000) \text{ , in degrees K}$$

For zone C, altitude “h” ranges from 20000 to 32000m:

$$Temperature = 228.65 + 0.001 \cdot (h - 32000) \text{ , in degrees K}$$

Pressure



For zone A, altitude "h" ranges from sea level to 11000m:

$$Pressure = 101325 \cdot \left[\frac{Temperature}{288.15}\right]^{5.1826} \quad \text{in Pascals}$$

For zone B, altitude "h" ranges from 11000 to 20000m:

$$Pressure = 22562 \cdot \left[\frac{Temperature}{215.65}\right]^{-307.43} \quad \text{in Pascals}$$

For zone C, altitude "h" ranges from 20000 to 32000m:

$$Pressure = 5441 \cdot \left[\frac{Temperature}{216.65}\right]^{-34.159} \quad \text{in Pascals}$$

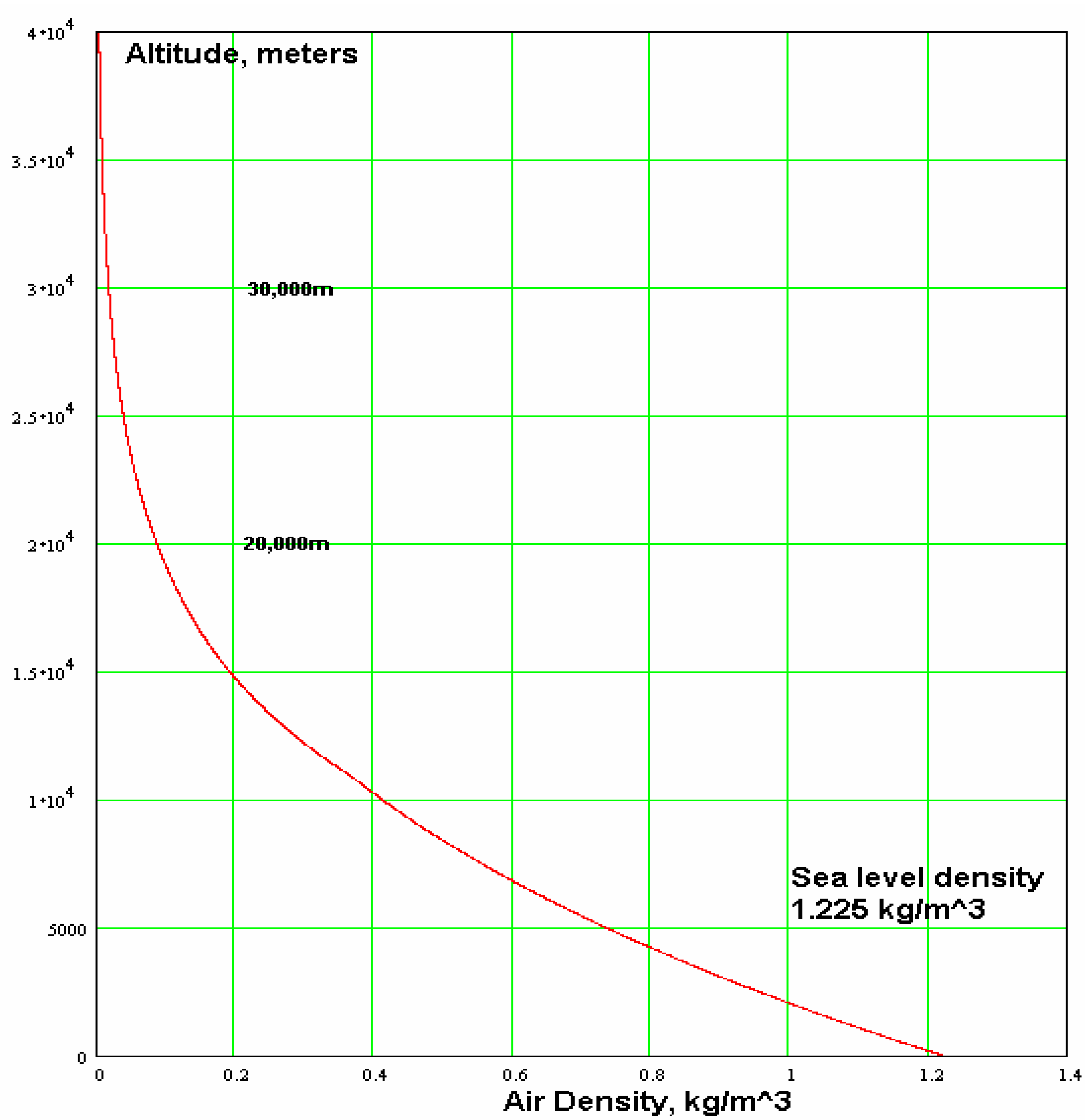
Use the appropriate temperature in deg K from calculated from the proper zone

Air Density

The air density is calculated with the ideal gas law:

$$AirDensity = \frac{Pressure}{287.1 \cdot Temperature} \quad \text{kilograms per cubic meter}$$

Where Pressure is in Pascals and Temperature is in degrees Kelvin



Approximation Formulas for Air Density

This is an approximation formula for air density as a function of height "h" in meters, good up to 23000m. It's near perfect up to 11000m, and then overestimates up to 12% at 17000m. Best to use it in the range of 0 to 11000m.

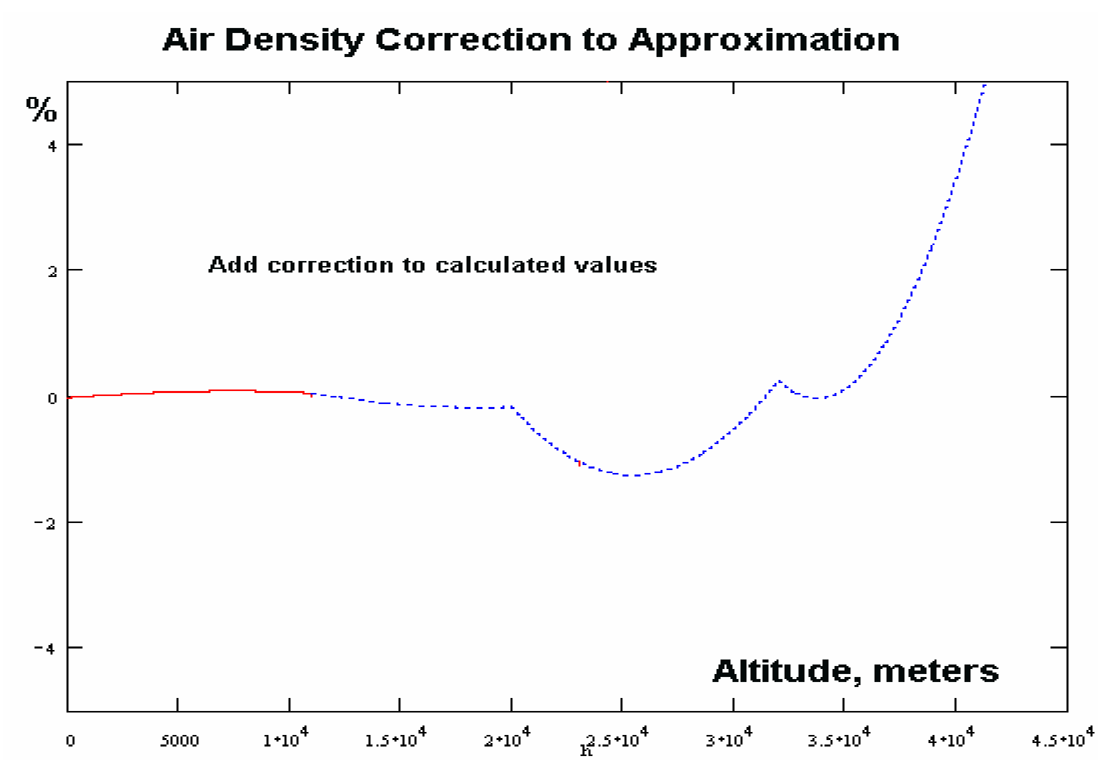
$$AirDensity_{0to11km} = 1.225 \cdot \left[1 - \frac{h}{44303}\right]^{4.25}$$

This one follows very closely through out the range from 20 to 42 km:

$$AirDensity_{11to42km} = 10.117 \cdot e^{-0.0001583 \cdot (h+10000)}$$

Maximum error ~ +1.3% at 26000m and ~ -5% at 42000m

Density in kilograms per cubic meter



Solar Montgolfiere Balloon Equations

A super temperature means that the air inside the balloon is hotter than the ambient air by ΔT degrees C. The air inside has slightly expanded, therefore less is dense due to its increased heat. It is this difference in ambient and lift gas densities that produces buoyancy lift.

The maximum mass is the total mass that the buoyancy force can lift with no free lift. In other words, it is the gross inflation divided by the acceleration of gravity. The total mass is composed of the suspended mass (everything that hooks onto the bottom of the balloon) + balloon envelope mass (lift gas mass is not necessary in these calculations). Of course, the mass of the displaced air is equal to the air density times the volume.

$$MaximumMass = Volume \cdot AirDensity \cdot \left[1 - \frac{Temperature}{Temperature + \Delta T}\right]$$

Temperature = air temperature at altitude “h”, degrees Kelvin
AirDensity = atmospheric density at altitude “h”, kg per cubic meters
Volume = balloon envelope volume, cubic meters (Note: cubic meters = cubic feet / 35.3)
ΔT = super temperature in degrees C, difference from ambient temperature
MaximumMass is in kg

The free lift is the net lift force, what’s left over from the buoyancy after lifting the suspended mass and the mass of the balloon envelope.

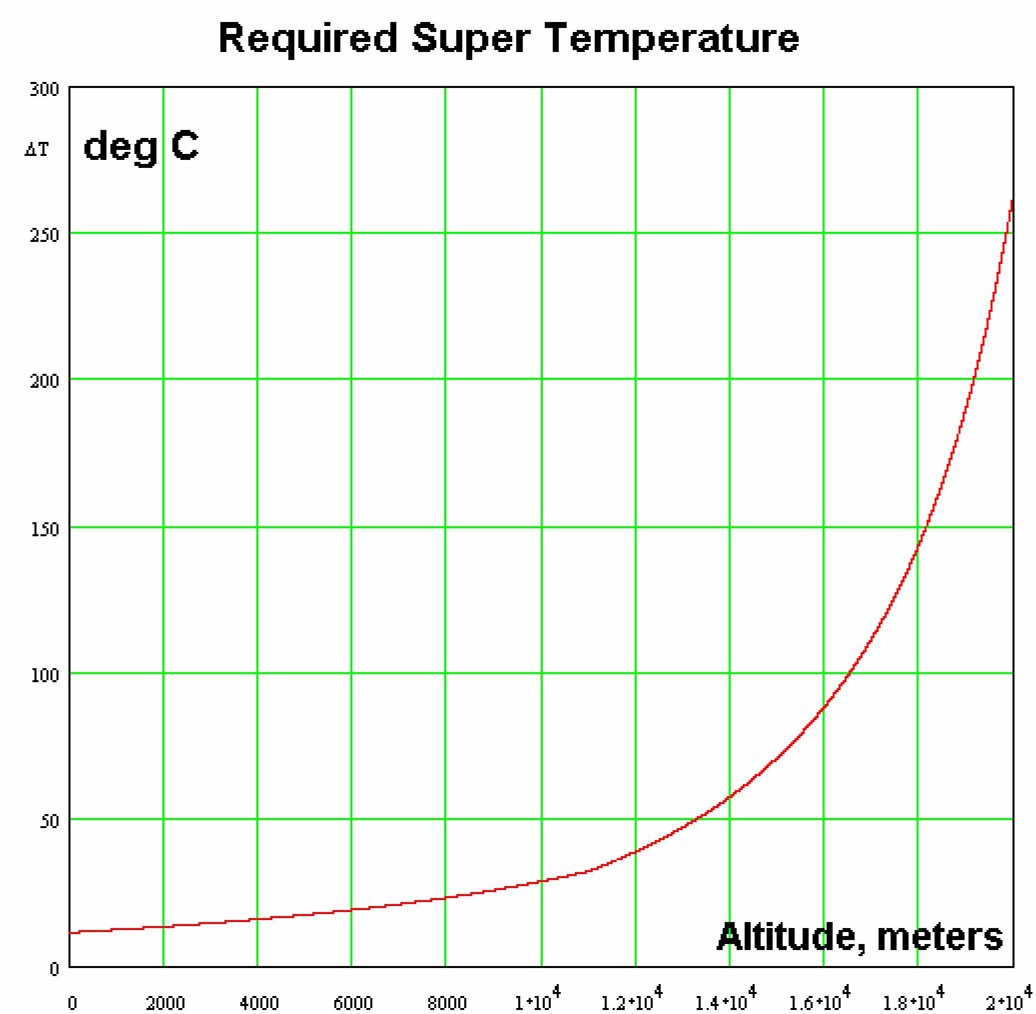
$$FreeLift = (MaximumMass - BalloonMass - SuspendedMass) \cdot g$$

The units for FreeLift is in Newtons (Note: lbs = Newtons/4.45)
g is the acceleration of gravity, = 9.807 m/s/s

If we solve the maximum mass equation for super temperature, we get this result:

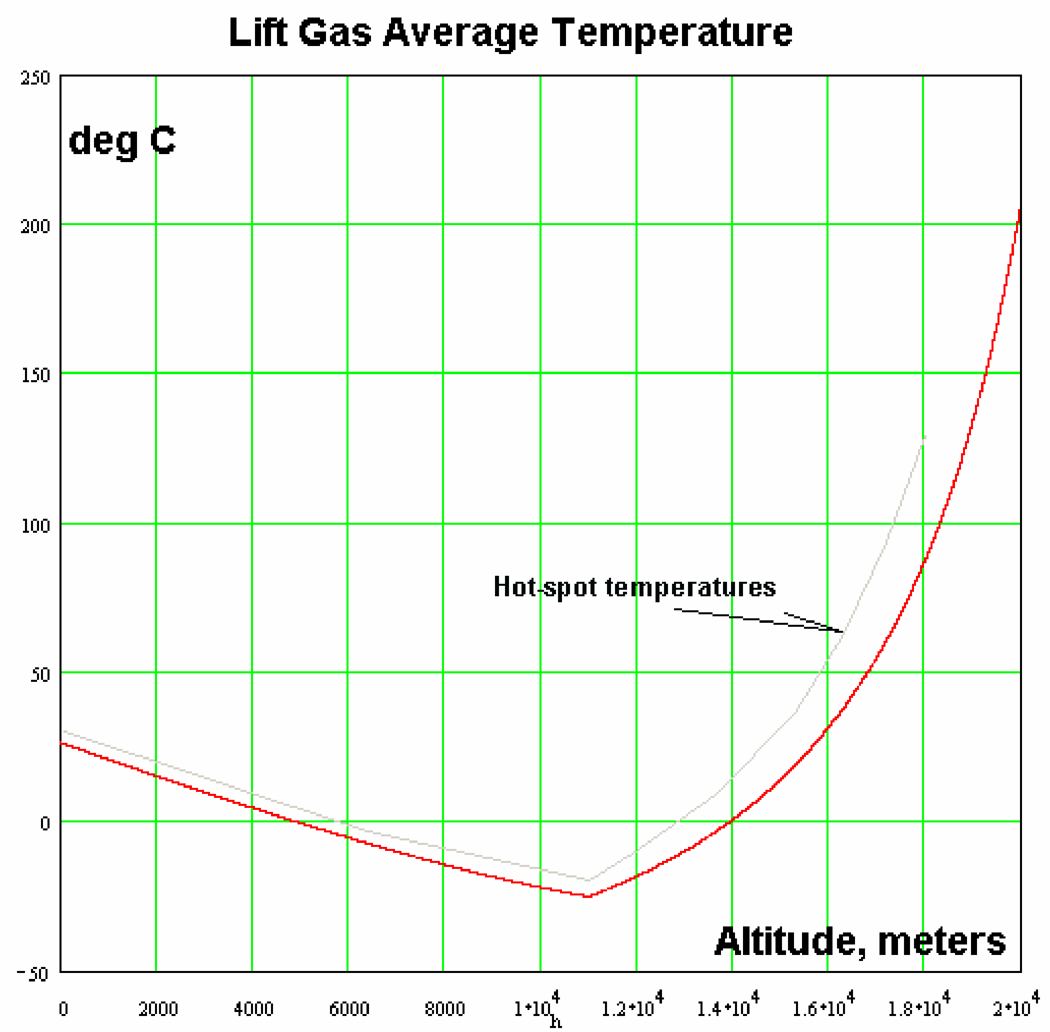
$$\Delta T = Temperature \cdot \left[\frac{1}{1 - \frac{MaximumMass}{Volume \cdot AirDensity}} - 1 \right] \quad °C$$

Where we use the same consistent units we have been using so far; kg, meters.
Using typical numbers that require ~ 12°C super temperature at launch, we can generate this curve of the required super temperature to attain buoyancy equilibrium.



The required super temperature jumps quite high after the tropopause running to unthinkable values into the lower stratosphere.

We can also express the above curve with the bulk average lift gas temperature, which is just adding the super temperature to the ambient air temperature. We can assume that the average skin temperature runs about the same as the bulk gas temperature. However, there are gradients, or hot and cold spots that determine the average. My estimates run as high as 35C hotter than the average which means that some parts of the skin are at ~ 100C when at 17000m. The polyethylene would be extremely susceptible to runaway creep at these temperatures, even with the lowest of stresses.



A solar Montgolfiere normally wants to fly up right up to the tropopause, and continue as far as it can until it just cannot keep up with the super temperature requirement. Unfortunately at those altitudes, its bulk temperature and the temperature gradients are greatest, leading to melt-hole failures.