

Dieluohan, a New Frontier for Multiple Sudoku Overlaps

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Abstract

The three classic Sudoku rules are modified slightly in this paper for Dieluohan study. By eliminating the numbering limitation rule on each 3X3 grid, and replacing it with a numbering limitation on each cell position for all its' 3X3 grids, we are able to construct a highly overlapped multiple layer Sudoku with 81 layers of superimposition. In this paper, Dieluohan degree and the Dieluohan pattern have been defined. There is no size limit for building very large Dieluohan due to its replicate nature.

Introduction

Dieluohan¹ is an overlap of multiple Sudokus. Dieluohan degree is the measurement of maximum overlaps of multiple Sudokus. Guo¹ has shown that if a 3X3 grid is the unit of superimposition, then the maximum number of overlapping Sudokus at any one unit is nine. With a smaller overlap unit, such as a single cell, the degree of superimposition grows. It is difficult if not impossible to arrange such a Sudoku structure with the rule that limits the numbering on each unit of 3X3 grid. Surrounding each such 3X3 grid with too many other similar 3X3 grids will provide too much strain for building such a Dieluohan.

Plum Sudoku

Rainbow Sudoku¹ or Disjoint Groups Sudoku, *e.g.*, grid A1~I9 in Table 1, has one more additional rule than Standard Sudoku. This added rule is that no number is allowed to repeat on the same cell position of every 3X3 grid. If we keep this rule along with the row and column limitations as the only three basic rules for Sudoku and making Dieluohan construction, then we can easily build a high degree Dieluohan, thanks to the dispersion characteristics of those restricted numbers. The Sudokus that follow these new rules we shall name Plum Sudokus.

A branch of Chinese Kung Fu named “Plum flower stepping stone technique” uses high raised stepping stones arranged in a pattern like plum flower petals for

practicing Kung Fu. The well-known two persons lion-dance performed in China uses similar technique in performing the dance. Two dancers wearing a lion costume coordinately climb several high raised stepping logs one by one to get the monetary reward kept in a red envelope hanging on top of a pole by a wealthy patron. Plum Sudoku presents a similar characteristic with this Chinese Kung Fu in that all its' associated numbers are arranged in an orderly dispersed pattern. An example of Plum Sudoku can be seen in Table 1, *e.g.*, the 9X9 grid of B1~J9. Due to its disjointed but orderly pattern, we are able to construct many high-leveled Dieluohans.

Definition

Before we proceed, the following terms need to be clearly defined:

Dieluohan:

A Sudoku structure made of several Sudokus. Each Sudoku overlaps with the other Sudokus.

Dieluohan degree (Dieluohan layers):

The maximum numbering counts of individual Sudokus overlapped in the same part of a Sudoku grid within a Dieluohan.

Dieluohan Pattern:

Patterns generated by combinations of Sudokus in a Dieluohan.

Example 1

Table1 is an 11X11 Dieluohan. It contains a total of nine Sudokus. A1~I9 is a Rainbow Web Sudoku¹, A3~K9 is a Rainbow Sudoku, and the other seven are all Plum Sudokus. The Dieluohan layer is determined by a total of nine degrees of superimposition on a shared 7X7 core grid, C3~I9. We therefore have nine degrees of Dieluohan in this case.

Example 2

Table 2 is another example of a super overlapped Dieluohan. In this 17X17 Dieluohan, a total of 81 Dieluohan layers have been found all containing the centered core cell I9. There are nine Rainbow Web Sudokus (A1~I9, D1~L9, G1~O9, A4~I12, D4~L12, G4~O12, A7~I15, D7~L15, and G7~O15) and 72 Plum Sudokus. This matrix reaches the ultimate possible number of 81 degrees of Dieluohan.

The Replicate Property

High degree Dieluohan presents an interesting replicate property. Table 1 and Table 2 are both examples of having the highest degree of Dieluohan for their sizes. The four-corner grids next to the shared core grid are the exact replica of themselves, *e.g.*, 2X2 grids of A1~B2, J1~K2, A10~B11, and J10~K11 in Table 1. The top and the bottom pairs (C1~I2 and C10~I11 in Table 1) as well as the left and the right pairs (A3~B9 and J3~K9 in Table 1) on the four sides of the shared core grid (C3~I9 in Table 1) also duplicate each other. This is the result of row and column regulations, which produce replicates due to each Sudoku having to share 64 (8X8 in overlapping diagonally) or 72 (9X8 in overlapping horizontally or vertically) numbers with each one of its neighbors. In Table 2, the four replicated corner-grid sizes increase to 8X8, *e.g.*, A1~H8, as the shared grid shrinks to a single cell, I9. The two pairs of duplicated grids reduce to single lines.

In large-sized high-degree Dieluohan, the Sudoku simply replicates itself in all directions. Since the basic grid for example 2 is only one 9X9 Sudoku, we can take advantage of this unique replicate nature to generate a super-giant Dieluohan quickly. The size can grow indefinitely.

The replicate characteristic also transforms Sudoku from an entertaining Mathematics game into a huge digital drawing board for artists.

Dieluohan Patterns

A big Dieluohan can have many smaller Dieluohans within its structure. It will be interesting to know how many smaller Dieluohans can be generated within a big Dieluohan. Figure 1 is a list of Dieluohan pattern shapes for example 1. We find it contains a total of 43 different patterns. With permutation (rotation, vertical/horizontal mirror) we find 222 different Dieluohan patterns. A smaller number of Sudokus can generate multiple patterns but with the same Dieluohan degree. As the Dieluohan matrix grows larger, so do the number of Dieluohan patterns within a Dieluohan.

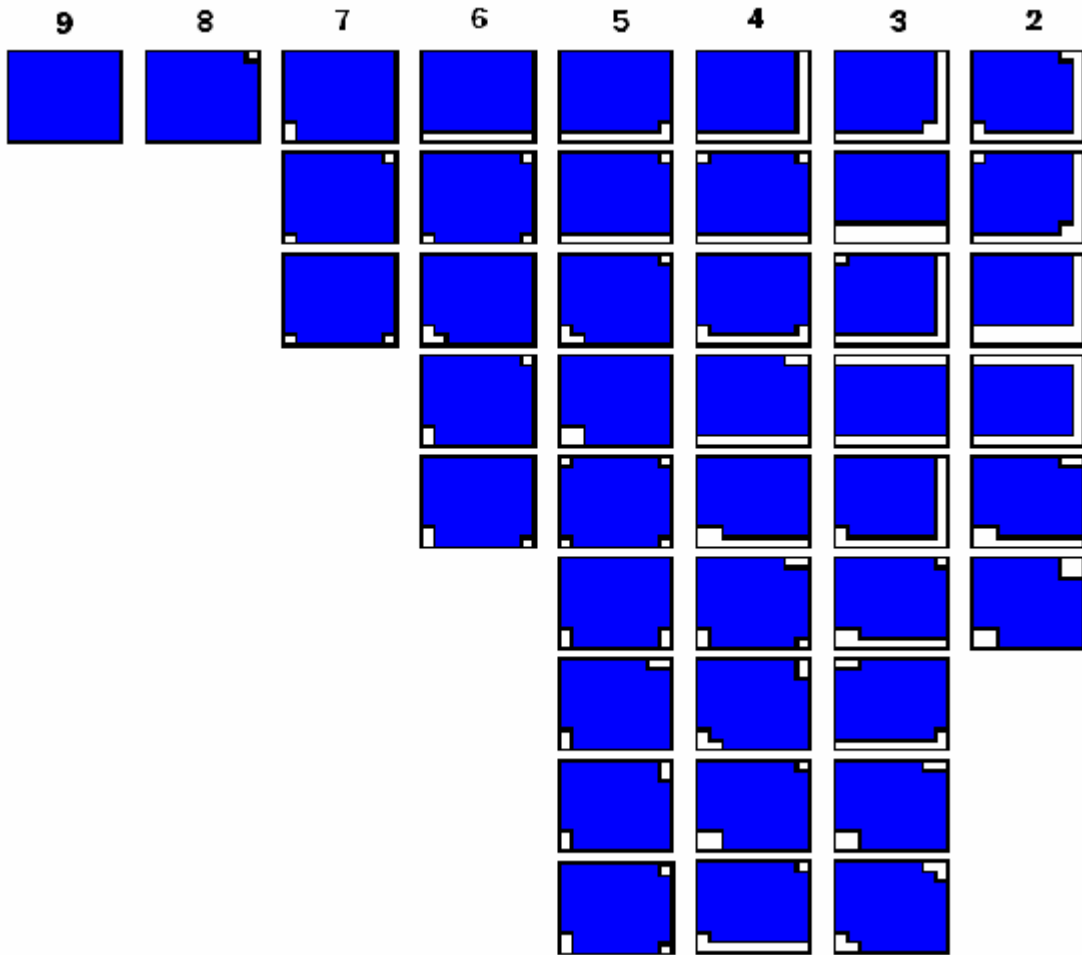
Table 1. A 9-degree Dieluohan table. In this diagram, a 7X7 shared core grid (blue) at the center, C3~I9, is shared by all 9 9X9 Sudokus. . The coordinates follow spreadsheet convention. Numerals represent the row number; and uppercased alphabets represent the column number. Lowercased alphabets represent any numerical number from 1 to 9 contained in a Sudoku puzzle.

	A	B	C	D	E	F	G	H	I	J	K
1	a	b	c	d	e	f	g	h	i	a	b
2	e	f	d	i	g	h	a	b	c	e	f
3	g	h	i	c	a	b	e	f	d	g	h
4	c	a	b	e	f	d	i	g	h	c	a
5	d	e	f	g	h	i	c	a	b	d	e
6	i	g	h	a	b	c	d	e	f	i	g
7	h	d	g	f	c	e	b	i	a	h	d
8	b	c	a	h	i	g	f	d	e	b	c
9	f	i	e	b	d	a	h	c	g	f	i
10	a	b	c	d	e	f	g	h	i	a	b
11	e	f	d	i	g	h	a	b	c	e	f

Table 2. An 81 layer Dieluohan table. In this diagram, a single cell at the center, I9 (blue), is shared by all 81 9X9 Sudokus.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1	a	b	c	d	e	f	g	h	i	a	b	c	d	e	f	g	h
2	h	i	g	a	b	c	e	d	f	h	i	g	a	b	c	e	d
3	d	e	f	h	i	g	c	a	b	d	e	f	h	i	g	c	a
4	e	f	d	b	c	a	i	g	h	e	f	d	b	c	a	i	g
5	b	c	a	i	g	h	d	f	e	b	c	a	i	g	h	d	f
6	i	g	h	e	f	d	b	c	a	i	g	h	e	f	d	b	c
7	c	a	b	f	d	e	h	i	g	c	a	b	f	d	e	h	i
8	g	h	i	c	a	b	f	e	d	g	h	i	c	a	b	f	e
9	f	d	e	g	h	i	a	b	c	f	d	e	g	h	i	a	b
10	a	b	c	d	e	f	g	h	i	a	b	c	d	e	f	g	h
11	h	i	g	a	b	c	e	d	f	h	i	g	a	b	c	e	d
12	d	e	f	h	i	g	c	a	b	d	e	f	h	i	g	c	a
13	e	f	d	b	c	a	i	g	h	e	f	d	b	c	a	i	g
14	b	c	a	i	g	h	d	f	e	b	c	a	i	g	h	d	f
15	i	g	h	e	f	d	b	c	a	i	g	h	e	f	d	b	c
16	c	a	b	f	d	e	h	i	g	c	a	b	f	d	e	h	i
17	g	h	i	c	a	b	f	e	d	g	h	i	c	a	b	f	e

Figure 1. Some Dieluohan shape-patterns for the 11X11 Dieluohan shown on Table 1. The numbers on top of the shapes are Dieluohan degree. Shapes with same Dieluohan degree are grouped vertically.



Acknowledgement

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Reference

1. Jau-Min S. Guo, "Dieluohan, the Superimposition of Multiple 9X9 Sudokus", <http://mysite.verizon.net/res7seqq/id15.html> (October 9, 2006).