

Dieluohan, the Superimposition of Multiple 9X9 Sudokus

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Abstract

Degrees of multiple superimpositions of many 9X9 Sudokus forming a tight ring structure is discussed. The degree of overall superimposition is the maximum number of the Sudokus that are superimposed on any one of the 3X3 square blocks in the ring. A new 9X9 Sudoku named Rainbow web Sudoku is introduced.

The Sudoku puzzle is a logic numbering game. Since it has no language barrier, it has become a worldwide phenomenon. Sudoku has a variety of forms. The most popular is the 9X9, 81-squared puzzle. The objective of the game is to fill up all squares with numbers from 1 to 9 while observing three very simple rules: for every row, column, and 3X3 squares unit, no repeated numbers can appear. The name Sudoku comes from a Japanese phrase which means “digit appears only once”. Sudoku probably originated from the ancient Chinese game named “Magic Squares”. It was first documented in the Arabia Encyclopedia in 990 known as “Latin Squares”. The modern version of the game was invented by Howard Garns in 1979 in the United States and was named “Number Place”. After 1986, the game fever started in Japan and Sudoku became the official name of the game.

As mention before, a classical or standard 9X9 Sudoku, e.g., the 9X9 square D1~L9 in Table 1, has only three simple rules.

1. No repeating number can appear on any row.
2. No repeating number can appear on any column.
3. No repeating number can appear on any of the nine individual 3X3 blocks.

A Sudoku with added diagonal restrictions, e.g., the 9X9 square D4~L12 in Table 1, has been named Diagonal Sudoku. It is also named X Sudoku because of its diagonal shape, or Web Sudoku because it is interconnected like a spider web. A Sudoku with added restriction of 3X3 square block positioning, that no repeating number can appear on the same position of any 3X3 square block, has been named Disjoint Groups Sudoku. An example is the 9X9 square G1~O9 in Table 2. It is also named Color Sudoku or Rainbow Sudoku due to the different colors on each of the individual nine positions on all 3X3 square blocks for easy viewing. A more complicated Sudoku with both diagonal

restrictions and 3X3 square block positioning restrictions added is named Rainbow Web Sudoku. An example is the 9X9 square G7~O15 in Table 2.

Dieluohan (pronounced Die-Lo-Han), in Chinese, is a show of acrobatics with many people forming a multi-layered human body pattern. A Sudoku can also be formed using a similar superimposed Dieluohan pattern if given enough numbers of Sudoku to play.

Puzzles with multiple 9X9 Sudokus have been created with the ability of overlapping. However, the superimposition of Sudoku has never been seriously explored beyond merely a two-layered 3X3-unit overlap. A cross-shaped four-ring Sudoku and Samurai Sudoku are few examples of this phenomenon. In this paper, more examples of multiple-layered superimpositions are discussed.

In the case of Samurai Sudoku, five standard Sudoku stacks can be superimposed to form a shape similar to a butterfly. This arrangement is a two-layered Dieluohan. A single centered Sudoku forms a layer by itself at the bottom, and the four remaining Sudokus form another layer all together at the top. The four Sudokus at the top layer are not connected or superimposed on each other; however, they each share one 3X3 square block with the bottom layered Sudoku. Samurai Sudoku is an example of two-layered Dieluohan. The degree of maximum superimposition for Samurai Sudoku is therefore two.

Dieluohan Sudoku can have many forms as well as layers. In addition to the many two-layered non-ring Sudokus, it can form a Sudoku ring. The degree of superimposition for a Sudoku ring can be higher, since a ring structure requires each Sudoku to share at least two 3X3 square blocks with others in order to form a ring.

Dieluohan Sudoku can be further classified as homogeneous and non-homogeneous. When all Sudoku members involved in construction of a Dieluohan Sudoku are of the same type; e.g., all are Standard Sudoku, then it is a homogeneous Standard Dieluohan Sudoku. Otherwise it is a non-homogeneous Dieluohan Sudoku.

Table 1 is a typical example of a non-homogeneous five-layered Dieluohan Sudoku ring. It contains four Standard Sudokus (D1~L9, A4~I12, G4~O12, and D7~L15) surrounding a centered Web Sudoku (D4~L12), which forms a tight ring. Each individual Sudoku shares the 3X3 square block in the center (G7~I9) of the puzzle with four other Sudokus. Five degrees of superimposition are easily determined. It is therefore a five-layered Dieluohan.

Table 2 is another example of non-homogeneous five-layered Dieluohan Sudoku ring. It is formed by a combination of a centered Rainbow Web Sudoku (G7~O15) and eight surrounding Rainbow Sudokus (G1~O9, G4~O12, G10~18, G13~21, A7~I15, D7~L15, J7~R15, and M7~U15). We counted the 3X3 square blocks that were shared with all 9X9 Sudokus and found the entire area of the centered Sudoku is shared by all

remaining four Sudokus. However, the maximum number of superimposition is five. It is only classified as a five-layered Dieluohan.

Table 3 is a typical example of homogeneous nine-layered Dieluohan Sudoku ring. It has nine Rainbow Web Sudokus forming a very compact 15X15-ring formation. All nine Sudokus (A1~I9, D1~L9, G1~O9, A4~I12, D4~L12, G4~O12, A7~I15, D7~L15, and G7~O15) share the same center 3X3 square block (G7~I9) thus having nine degrees of superposition. It is therefore a nine-layered Dieluohan Sudoku.

In both examples shown in Table 2 and Table 3, we can also find an imbedded Dieluohan Sudoku ring like that shown in Table 1. A larger-sized and higher-degreed Dieluohan Sudoku ring can have a smaller-sized and lower-degreed Dieluohan Sudoku ring imbedded within.

The more compact the Dieluohan structure, the higher the number of Dieluohan layers it can have. The example in Table 1 contains five Sudokus. It involves the same amount of Sudokus as a Samurai Sudoku; yet it produces more Dieluohan layers. More shared 3X3 square blocks results in higher degrees of superimposition, and hence smaller sizes. We can also compare the examples in Table 2 and Table 3. A more compact square-shaped Sudoku ring forms more Dieluohan layers than less compact cross-shaped Sudoku ring.

A 9X9 Sudoku has a total of nine 3X3 square blocks. If the 3X3 square block is taken as the basic unit for the superimposition, then no more than nine different Sudokus can be superimposed. Therefore, nine is the highest possible number of Dieluohan layers superimposed. If this restriction is relaxed to include smaller units, the number of Dieluohan layers increases.

Sudoku's magic is truly more than just its two-dimensional stunning display. The ability of its superimposition is also very interesting. Different shaped and larger sized Dieluohan Sudokus can only be limited to our imagination.

Table 1. A non-homogeneous five-layered Dieluohan Sudoku ring. This ring contains one 9X9 Web Sudoku at the center and four 9X9 Standard Sudokus at the four sides. The coordinates follow spreadsheet convention. Numerals represent the row number; and uppercased alphabets represent the column number. Lowercased alphabets represent any numerical number from 1 to 9 contained in the Sudoku puzzle.

	A	S	C	D	E	F	G	H	I	J	K	L	M	N	O
1				a	b	c	d	e	f	g	h	i			
2				e	f	d	i	h	g	b	a	c			
3				g	h	i	c	a	b	f	d	e			
4	i	b	e	d	g	h	a	f	c	i	e	b	d	h	g
5	g	a	f	c	e	b	h	i	d	a	g	f	e	b	c
6	c	d	h	i	a	f	g	b	e	d	c	h	f	i	a
7	f	g	c	h	i	e	b	d	a	c	f	g	i	e	h
8	a	h	b	f	d	g	e	c	i	h	b	a	g	d	f
9	e	i	d	b	c	a	f	g	h	e	i	d	c	a	b
10	d	e	g	a	f	i	c	h	b	g	d	e	a	f	i
11	b	c	a	g	h	d	i	e	f	b	a	c	h	g	d
12	h	f	i	e	b	c	d	a	g	f	h	i	b	c	e
13				c	g	f	h	i	d	a	e	b			
14				i	a	h	g	b	e	d	c	f			
15				d	e	b	a	f	c	i	g	h			

Table 2. A non-homogeneous five-layered Dieluohan Sudoku ring. There are five 9X9 Sudokus evenly lined up in both directions of the cross-shaped structure. At the center is a Rainbow Web Sudoku. All eight others are Rainbow Sudokus.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
1							a	b	c	d	e	f	g	h	i						
2							f	h	i	g	c	b	a	d	e						
3							g	d	e	h	i	a	f	b	c						
4							h	c	g	f	d	e	b	i	a						
5							d	i	a	c	b	g	h	e	f						
6							b	e	f	a	h	i	d	c	g						
7	f	e	d	c	g	b	i	a	h	e	f	d	c	g	b	i	a	h	f	d	e
8	b	g	c	i	a	h	e	f	d	b	g	c	i	a	h	e	f	d	b	c	g
9	i	h	a	e	f	d	c	g	b	i	a	h	e	f	d	c	g	b	i	h	a
10	e	d	f	a	h	i	g	b	c	d	e	f	a	h	i	b	c	g	e	f	d
11	g	c	b	d	e	f	a	h	i	g	c	b	f	d	e	h	i	a	g	b	c
12	a	i	h	g	b	c	f	d	e	h	i	a	g	b	c	f	d	e	a	i	h
13	d	f	e	h	i	a	b	c	g	f	d	e	h	i	a	g	b	c	d	e	f
14	c	b	g	f	d	e	h	i	a	c	b	g	d	e	f	a	h	i	c	g	b
15	h	a	i	b	c	g	d	e	f	a	h	i	b	c	g	d	e	f	h	a	i
16							c	f	h	i	a	d	e	g	b						
17							e	a	d	b	g	c	i	f	h						
18							i	g	b	e	f	h	c	a	d						
19							a	b	c	d	e	f	g	h	i						
20							f	h	i	g	c	b	a	d	e						
21							g	d	e	h	i	a	f	b	c						

Table 3. A Rainbow Web homogeneous nine-layered Dieluohan Sudoku ring. Nine 9X9 Rainbow Web Sudokus are evenly distributed on this 15X15 square matrix.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	a	b	c	d	e	f	g	h	i	a	b	c	d	e	f
2	i	g	h	b	c	a	d	e	f	h	g	i	b	c	a
3	f	d	e	g	h	i	b	c	a	f	d	e	g	h	i
4	c	a	b	h	i	g	e	f	d	c	a	b	h	i	g
5	h	i	g	e	f	d	c	a	b	g	i	h	e	f	d
6	e	f	d	c	a	b	h	i	g	e	f	d	c	a	b
7	b	c	a	f	d	e	i	g	h	b	c	a	f	d	e
8	g	h	i	a	b	c	f	d	e	i	h	g	a	b	c
9	d	e	f	i	g	h	a	b	c	d	e	f	i	g	h
10	a	b	c	d	e	f	g	h	i	a	b	c	d	e	f
11	i	g	h	b	c	a	d	e	f	h	g	i	b	c	a
12	f	d	e	g	h	i	b	c	a	f	d	e	g	h	i
13	c	a	b	h	i	g	e	f	d	c	a	b	h	i	g
14	h	i	g	e	f	d	c	a	b	g	i	h	e	f	d
15	e	f	d	c	a	b	h	i	g	e	f	d	c	a	b