

San, Chuan, and Super Constrained Sudokus

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Abstract

Properties of San and Chuan in a Sudoku are explained. The discovery of Jia Zi Sudoku and Fu Xi Sudoku are two milestones for super constrained Sudokus. Jia Zi Sudoku contains a full range of Box, Plum, San, and Chuan properties. Fu Xi Sudoku is derived from 9X9 Magic Square that is from the Holy Turtle Magic Square class. Fu Xi Sudoku contains only San and Chuan properties, but has far more 9-digit constraints.

San and Chuan Sudokus

San and Chuan Sudokus^{1,2} are recently found and recognized as two different forms of Sudoku. These Sudokus can actually be obtained by simple mathematics transformation¹. “San” is the Chinese character that means “three” which consists of three horizontal lines as a set and each set contains nine different digits (Figure 1). “Chuan” is a Chinese character that means “river” which consists of three vertical lines as a set and each set also contains nine different digits (Figure 2). These ordered constraint patterns are evenly distributed in a 9X9 Sudoku. These two types of Sudoku can both replace the nine 3X3 box constraints from an ordinary Standard Sudoku with their own nine distinguished sets of constraint and form their own Sudoku families.

Jia Zi Sudoku and its rules

Since the discovery of Plum Sudoku^{3,6} (Figure 3) and subsequently San Sudoku and Chuan Sudoku, a common Sudoku is no longer confined with the rule that requires nine 3X3 box grids (Figure 4). The discovery of new Sudokus that contain combinations of all these four constraints is just a matter of time.

Jia Zi Sudoku⁴ (Figure 5) is the first highly constrained Sudoku with sixty constraints, all involved with nine different digits. It contains all constraints from the boxes, the Plum groups, the San groups, and the Chuan groups.

The rules of Jia Zi Sudoku are listed as follows.

1. Nine 9-different-digit rows.
2. Nine 9-different-digit columns.
3. Nine 9-different-digit Sans.

4. Nine 9-different-digit Chuans.
5. Nine 9-different-digit 3X3 boxes.
6. Nine 9-different-digit Plums.
7. Every diagonal starts from the upper corners of a 3X3 box that contains 9 different digits.

The last rule requires a concept that a diagonal line which is shorter than nine cells can be continued at the other end of the Sudoku until nine members are counted. It looks like a diagonal line that was broken into two segments and placed on two ends of the 9X9 grids. This concept is nothing new since through a Dieluohan expansion^{5,6,7} the discontinued diagonal lines are recognized as continued. Let us give these kinds of discontinued diagonal lines a name, the “pseudo diagonals”. Rule number seven contains six diagonals: two ordinary diagonals and four pseudo diagonals. We therefore have a total of sixty 9-different-digit constraints for a Jia Zi Sudoku.

Jia Zi in Chinese means sixty years. “Jia” is the first of the ten numbers we call heavenly stems, Tian Gan. It is similar to A in a group from A to J in the alphabet. “Zi” is the first of the twelve zodiac symbols or Di Zhi, which represents rat. Zi is also used for counting time from 11 p.m. to 1 a.m., the first two-hour period in a day. A full day is divided into twelve time periods. The counting of the Chinese year started with “Jia Zi”. Every year the level moved down one unit and simultaneously zodiac moves down one unit. After sixty years, the cycle moves back to Jia Zi again. The significance of this type of timing system is somehow related to the vision of the earth as the center of our universe relative to the two brightest and largest objects in the sky, the sun and the moon. It was noticed since ancient times that nature presents a rhythm and cycle related to these numbers.

The sixty 9-different-digit constraints of the Sudoku mentioned earlier matches nicely with the name Jia Zi thus it was named Jia Zi Sudoku.

Without the help of a computer, it is difficult to imagine that Jia Zi Sudoku could exist. As a matter of fact not only does it exist; it actually has far more solutions than one can imagine. With sixty 9-different-digit constraints, a Jia Zi Sudoku has enough freedom to change and not to be confined to only one single solution.

A particularly useful method of deriving another Jia Zi Sudoku from an existing one is by applying Dieluohan Big Bang Theorem⁵. One can quickly produce many other combinations through Dieluohan grid expansion. Random number replacement is another way of generating Jia Zi Sudokus. There are other methods such as simultaneously exchanging six columns or six rows or both so that after the exchange no rule was broken. Of course, simple two-dimensional or three-dimensional rotations along symmetry axes will do the same.

Fu Xi Sudoku and its rules

A super constrained Fu Xi Sudoku⁸ (Figure 6) with numerous 9-digit constraints is another new discovery. Fu Xi Sudoku is a modification of a 9X9 Magic Square matrix that belongs to Holy Turtle Magic Square class¹⁰. The construction of a Fu Xi Sudoku is relatively easy by following a simple replacement rule on a 9X9 Holy Turtle Magic Square. If we place all digits with only 1 to 9 sequentially and repeatedly while filling up the 9X9 Holy Turtle Magic Square matrix we end up with a Fu Xi Sudoku.

Fu Xi Sudoku inherits most of the properties of a 9X9 Holy Turtle Magic Square¹⁰. Its rules are listed as follows.

1. Nine 9-different-digit rows.
2. Nine 9-different-digit columns.
3. Nine 9-different-digit Sans.
4. Nine 9-different-digit Chuans.
5. Nine 9-different-digit diagonals or pseudo diagonals parallel to each other.
6. Nine single-digit diagonals or pseudo diagonals each with nine members and perpendicular to the 9-different-digit diagonals.
7. The nine 9-different-digit diagonals and pseudo diagonals should be arranged in a fashion of cycled descending or ascending sequence.
8. Any 9-digit shaped pattern with a center point of symmetry has a sum equal to the sum of any 9-different-digit diagonal.

Fu Xi Sudoku belongs to the San and Chuan Sudoku families. It does not have a full range of nine 9-different-digit Box groups or Plum groups. It is thus not an ordinary Standard Sudoku or a Plum Sudoku. Fu Xi Sudoku does not involve as many orderly 9-different-digit constraints as Jia Zi Sudoku. Many of the 9-digit constraints of Fu Xi Sudoku contain less than nine different digits. Fu Xi Sudoku also has one distinct line of symmetry along one of its diagonals. This unique class of Sudoku seems like a rotating top or planet. There is currently another example of this class of Sudoku named Spinning Top Sudoku⁸ that has been published.

According to rule number 8, Fu Xi Sudoku has many 9-digit constraints with sums all equal to 45. The total possibilities of combinations of rule number 8 is $40C_4$, where 40 is the total number of available grids and C_4 is to choose only 4 of them. This is under the condition when the center digit is always present. The number of this combination is 91390.

From the other seven rules, in the point of view that includes 9-different-digit grids but not having center of symmetry, we have eight more 9-different-digit grids each coming from row groups, column groups, San groups, Chuan groups, and diagonals. There are additional 9-digit grids, but these involve less than nine different digits and have no center of symmetry: two 3X3 boxes from the Box group and two Plum sets from the Plum group all lying on the principal diagonal that is perpendicular to the spinning diagonal. Thus the total number of sums that equal 45 is calculated to be 91434.

Fu Xi Sudoku does not have many varieties due to loaded constraints. Once rule number 7, descending or ascending order, is applied for an axis there is only one solution that can

fulfill all the conditions. In other words, a Fu Xi Sudoku requires no clue to construct. Only four Fu Xi Sudokus exist.

King Fu Xi designed Bagua five thousand years ago with only eight Guas or eight trigrams in his diagram. A trigram consists of three lines, either a solid line or a broken line. The three lines represent heavenly sky, motherly earth, and human, the three most important elements in human society. It was later in Yi Jing when sophisticated 64-paired trigrams or hexagrams appeared. Each hexagram is also called a Gua. Bagua and Yi Jing have been used to predict the future and explain almost every natural phenomenon. It is interesting to note that there are nine Qian Guas in the San and Chuan Sudoku families together. A combination of San and Chuan resembles a “Qian Gua”, the Gua with a pair of solid trigrams, which means the best of the best. The name of Bagua Sudoku had been taken for Sudoku with an octagon shaped figure⁹, thus Fu Xi is chosen in this special super constrained Sudoku which is also somewhat related to Bagua.

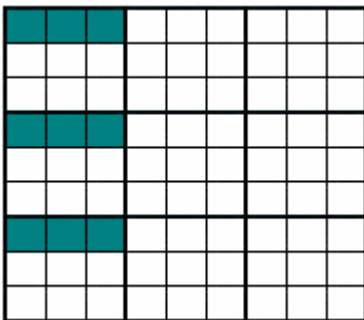


Figure 1, simplified graphic representation of a San Sudoku. The three darkened areas are shaped like the Chinese character “three” and are grouped as a set. San Sudoku has nine sets of San. San is shaped like a solid trigram.

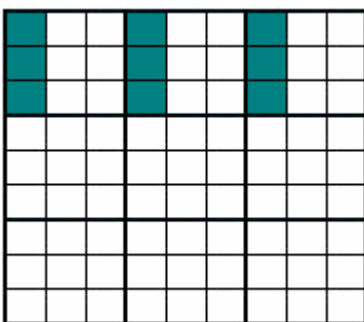


Figure 2, simplified graphic representation of a Chuan Sudoku. The three darkened areas are shaped like the Chinese character “river” and are grouped as a set. Chuan Sudoku has nine sets of Chuan. Chuan is also shaped like a solid trigram.

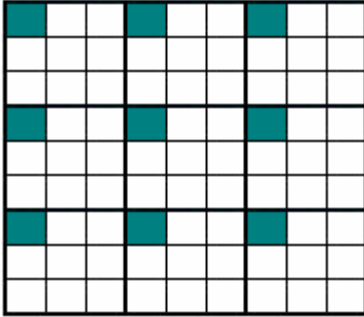


Figure 3, simplified graphic representation of a Plum Sudoku. The nine darkened spots are grouped as a set. Plum Sudoku has nine sets of Plum.

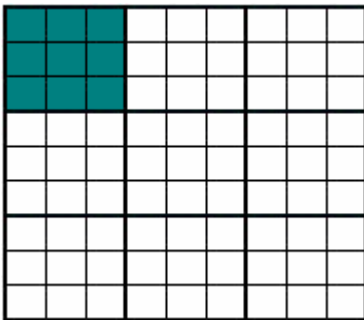


Figure 4, simplified graphic representation of an ordinary Standard Sudoku. The darkened box areas represent a set. The ordinary Sudoku has nine sets of 3X3 box.

	A	B	C	D	E	F	G	H	I
1	1	2	3	4	5	6	7	8	9
2	8	6	7	2	9	1	5	3	4
3	9	4	5	3	7	8	6	1	2
4	5	9	4	8	3	7	2	6	1
5	3	1	2	6	4	5	9	7	8
6	7	8	6	1	2	9	4	5	3
7	6	7	8	9	1	2	3	4	5
8	4	5	9	7	8	3	1	2	6
9	2	3	1	5	6	4	8	9	7

Figure 5, a 60 9-digit constrained Jia Zi Sudoku presented in a spread sheet arrangement.

	A	B	C	D	E	F	G	H	I
1	1	6	2	7	3	8	4	9	5
2	6	2	7	3	8	4	9	5	1
3	2	7	3	8	4	9	5	1	6
4	7	3	8	4	9	5	1	6	2
5	3	8	4	9	5	1	6	2	7
6	8	4	9	5	1	6	2	7	3
7	4	9	5	1	6	2	7	3	8
8	9	5	1	6	2	7	3	8	4
9	5	1	6	2	7	3	8	4	9

Figure 6, a super constrained Fu Xi Sudoku presented in a spread sheet arrangement.

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