Some Observational Notes

PWCS Math Workshop

***"Part II of the Understanding Math Series – Multiplication and Division"* 19 Feb 2009**

I had the pleasure of attending a PWCS Math Investigations (MI) Workshop last week presented by the PWCS Math Department leads. The presentation was run by Carol Knight and Linda Zoborofsky (Linda did most of the presentation; Carol ran the slides off the computer and provided supportive commentary throughout). The workshop stated goals were:

- *to understand what it means to learn multiplication and division for understanding*
- *to understand how multiplication and division facts can be learned in an engaging manner through the use of reasoning and games*
- *To learn how to help children become proficient with multiplication and division*
- *To recognize that children need access to a variety of strategies that lead to computational fluency with numbers*

The following are my observations on the evening. Hope you find them useful.

12 attendees had signed up in advance and their names were on the official roster. Since I was a "phone in add on" I wasn't on the roster with those 12 but signed in at the door. As for who showed up: 1 x mom and son (~4th grade I believe not sure which school); 1 x couple and their two daughters (one 5th, one 3d - Signal Hill Elementary school - so one's traditional [Grade 5] and the other's in MI); 1 x mom & 3d grade daughter (Fannie Fitzgerald Elem). Elizabeth Martinez – PWCS Director of Student Learning and Professional Development and Gail Hubbard - PWCS Gifted Education Supervisor just happened to be there as well. I'd run into Gail coming in the door and asked her if she was going to math night -- she said no that she had to prepare a lesson plan...but curiously opted for the Math Workshop. Elizabeth's daughter (4th grade?) was also there. I didn't ask either official if they were told to be there because I was in attendance, but I can't help but think might have been the case. Regardless, I think the staff didn't really know how to take my presence since I was very gracious and participated just like everyone else. It was a nice change from the antagonism I've had at Board meetings; think it was a pleasant meeting for all of us.

The evening was supposed to be a two hour session on both multiplication and division - but it took 85 minutes to get through the laborious examples of how to do multiplication - 1 digit by 1 digit through 2 digit by 3 digit…all by avoiding the use of traditional algorithms. I felt really bad for the Fannie Fitz mom and her daughter (I was sitting next to both of them). My sense is that the mom was there to try to figure out how to help her daughter – who she said has been struggling with the subject - lots of tears around the dinner table (*she told me*). The mom was trying to do the right thing and help her daughter by attending the math night. I don't think it was overly helpful; after the meeting I tried to encourage her to stick with the basics and recommended some online resources.

The session launched into multiplication - array cards called "algebraic flash cards;" moved on to "what fact could you use to find 9 x 3" (c*aught myself thinking, how about simply use the fact 9x3=27?*). It was explained that the array grid flash cards were superior "algebraic flash cards" because, as Linda stated, "*With traditional flash cards you either know it [the fact] or you don't*." -- but MI kids, "*Use what they know to find out what they don't know*." All well and good, but before MI, kids were somehow able to learn the basic facts and not have to constantly re-figure out what the basic facts are over and over again. I'll note up front that the evening had a lot of "infomercial" qualities and was peppered throughout with quirky MI proclamations and lamenting over the evils of traditional arithmetic algorithms. The flash card portion of the evening ended with a great quote from Linda, "[Investigations games] are more engaging; kids actually ask to do them in recess!" Again, an infomercial; for the record my kids never once have asked to stay in from recess to continue mathematics games in class. From there, the session moved on to multi-digit multiplication. Let me just say that the evening was peppered throughout with assertions that the traditional algorithms are simply shortcuts and "digit-based" where the place value is masked -- just shortcuts. I really do not believe our Math Department knows just what an algorithm really is. A repeated criticism of MI in national reviews of the curriculum is that the program does not lead to fluency with the standard arithmetic algorithms. Yet here I was listening to a discussion about just how bad these were for children.

The main 2-digit multiplication problem examined was 21 x 16 – stacked vertically in the example. The presentation went through multiple, multiple "strategies" for solving this. Participants were asked to work the problems on paper using the "MI way" then given the opportunity to share solutions on the board. A classic moment was when one of the children attempted the MI partial product "strategy" at the board but got confused so Linda completed it for her. Interestingly enough when lamenting the pitfalls of the traditional algorithm they used the 21 x 16 case as an example. The criticism was that when stacking 21 on top of 16 first you multiply the 6 x 21 and get 126 then below you "*shift over one space to the left*" and multiply "2 x 16". They didn't put a 0 in the ones place. The assertion is that this is all teachers teach when presenting the traditional algorithms – "*shift over one space.*" It was asserted that this is only "digit moving" and that children just can't learn this…and apparently PWCS teachers don't understand it and aren't able to teach it either.

At the Q&A session at the end I said I was surprised that our teachers weren't able to articulate how the traditional algorithm worked and explain to children how to succeed using this method that's been successfully taught to generations of children worldwide. Linda and Carol explained that, "Our teachers were never taught that way," and they're, "Only doing what they were taught." I didn't get into a debate but did say that in my experience living in and participating in school districts all across the country (and around the world for that matter) this is the first place I've ever been where it appears we must have a pocket of teachers who aren't able to understand and instruct the concept and application of the traditional algorithms.

In any case, we moved on through all the MI strategeries for multiplication. It was an interesting exercise in making mathematics unnecessarily complicated. You have to set aside what you know about simple computational skills to do arithmetic using these strategies – it helps though approaching the problem as an adult with a lifetime of mathematical experience under your belt. Children on the other hand – not so much. It was interesting that Carol asserted that using MI partial product strategy was especially useful when children get up to “three digit by three digit” multiplication because using the traditional algorithm, “*They can’t keep up with the carrying so partial products are easier to keep track of.*”

Try multiplying 123 x 456 with partial products. You end up with a column of 9 addends (*if kept track of properly*). You also have to “keep track of” whether or not the *single digits* you are multiplying together are ones, tens, or hundreds and recognize that the partial products of these may actually be ones, tens, hundreds, thousands, or tens of thousands. And *single digits* is key to this. It’s like pre-arithmetic where the concept is to confine the arithmetic to one digit by one digit multipliers. As for whether or not that’s a one multiplied by a ten or hundred or thousand? Well, I guess you can just teach them to “shift over one or two spaces to the left.” Which, given what was said earlier about the PWCS teachers capabilities I’d bet that’s what’s being “taught.” Conceptually the standard application a higher order function as it exercises the students’ knowledge of any/all of the basic multiplication facts and combines this with the “number sense” of the place values (ones, tens, hundreds, etc.).

(MI)	
	1 1
	11
	1 1
123	123
x456	x456
18	738
120	6150
600	+ 49200
150	= 56,088
1000	
5000	
1200	
8000	
+ 40000	
= 56,088	**(Standard)**

OK, admittedly, I had initial difficulty "keeping up" with the long columns of partial products and screwed it up once. Of course, thankfully I could whip through the traditional algorithm and catch my errors.

The presentation then went through the "binomial" break apart strategy that's supposed to be teaching kids "algebraic principles in 4th grade", the "change it and make it easier problem" using 98 x 15 as an example (ok change it to 100x15 - 2x15 = 1500 – 30 = 1470). This is something that many adults gravitate towards – again given more than 8-9 years of experience, but it's not something that's overly intuitive; particularly to small children trying to get a foundation of basic arithmetic. I really got a chuckle out of Gail Hubbard's approach – she "changed it" to 100 x 13 and got completely, irreversibly lost. I guess some school administrators just shouldn't "*do*" math. Imagine now if you will an 8 or 9 year old child struggling with the same issues…

The session then moved on to the "strategy" of "area model - open array, partial product" multiplication of two digit numbers. This one was said to be superior because it teaches children "algebra" in 4th grade. The example given was 78 x 35 for fourth grade. For the "open-array partial product area model strategy" (let's just use a *shortcut* for the sake of call it the "*OAPPAM*" strategy). For the OAPPAM strategy one must break both numbers into parts 78 = 70 + 8; 35 = 30+5. Then you make a 2x2 box/matrix and on the outside at the top of the matrix place 70 above the upper left quadrant and 8 above the upper right quadrant. Down the left side of the outside of the matrix you place 30 adjacent to the upper left quadrant and 5 adjacent to the lower left quadrant. Then you multiply the partial products: 70x30=2100 (placed in upper left quadrant of the matrix) 30 x 8 =240 (placed in upper right quadrant), 5 x 70 = 350 (placed in lower left quadrant), and 5 x 8=40 (placed in lower right quadrant). Then one writes the string of partial products below the matrix thusly: 78x35= (70+8)(30+5) now, just how 4th graders know that one doesn't need to have a "x" sign between the two parenthesized terms is a stretch, but hey, it is deeper math...And finally below that one writes: 2100 + 350 + 240 + 40 = 2730. Can this lead to a correct solution? Of course. Is it appropriate for 4th graders…given that one is not teaching them standard algorithms? Hardly. I wanted to ask how children were supposed to use OAPPAM with decimal multiplication for 7.8 x .35 but we'd already exhausted 85 minutes of the session and I was anxious to get on with division.

	70	8
30	2100	240
5	350	40

78 x 35 = (70+8) (30+5)
2100 + 350 + 240 + 40 = 2730

My take away for the multiplication portion of the workshop: if PWCS teachers (or math office for that matter) are unable to learn and comprehend how to instruct the traditional algorithms how on earth are they going to be able to explain simple multiplication using open array partial products and binomial expressions to 8-11 year olds? By the end of the multiplication session the little girl who been struggling had lost interest and wandered around a bit. And though Linda and Carol asserted that the traditional algorithm is "*taught*" in 5th grade, Linda had this to say about the traditional multiplication algorithm, "*You [children] can use any method, but you have to explain how you did what you did*." The point of mathematics is that the correct and proper application of the algorithm *is the explanation*. Solving 21 x 16 or 35 x 78 with the traditional algorithm *explains* the solution:

(Standard Algorithm)	
	3
	4
21	35
x16	x78
126	280
+210	+2450
= 336	= 2730

From there we went through a couple of examples of "Multiplication in Algebra I" (2a+8)(3a+5) and using the MI OAPPAM strategy described previously one gets described above one gets $6a^2+34a+40$. What they were trying to convey to the audience is the superiority of teaching algebra to 4th graders by having them learn/memorize a procedure that is applicable to Algebra I binomial solutions. As I watched the parents in the audience my sense was that they were looking for some reason to believe in this approach and since Linda and Carol said it helps with Algebra, they were accepting it at face value. Classic quote from Carol was, "*I'll bet my paycheck that if they can do this in 4th grade they can do this in Algebra I.*" I guess I'd bet my paycheck that if teachers are unable to learn and teach the standard basic arithmetic algorithms they don't stand a chance with binomial problems. And remember, it wasn't me who said our teachers don't understand traditional algorithms. Perhaps some of the allotted $150,000 2008-2009 PWCS mathematics professional development funds[1] slated for continuous improvement of elementary mathematics knowledge ought to be set aside to provide training in comprehension and teaching of traditional algorithms. I'd be happy to conduct classes for say $500.00 per class.

[1] PWCS Memorandum from Kris Pedersen, Interim Associate Superintendent, Student Learning and Accountability to Dr. Steve Walts, 17 Feb 2009

Around 8:35 p.m. we "moved on" to division. Spent 15 minutes showing just how difficult we can make it to have 3d-5th graders solve 56 divided by 4. Nowhere is the universal algorithm even "studied" in Investigations - nowhere at all and Linda and Carol were very clear about this. Instead children are left with "strategies" that are interestingly tortuous – to adults. Admittedly after I cleared my mind of simple approaches to division, I was able to "master the MI techniques" but I will be brutally honest, if you're not teaching your kids at home how to do long division - starting in 3d grade (*like most kids in the US are allowed to start doing so*) it's just going to reduce the competition in the future job market for those kids whose parents are doing so. Again, the "division" part of the evening was rushed and never got the attention that the multiplication part did. And truthfully, division will kill kids if it is "taught" under the MI umbrella. It remains like so much of MI computational instruction – at the "pre-algorithm / pre-arithmetic" level of learning. So make sure you're doing what you need to at home to teach your children. I think Gail Hubbard summed it up best by stating, "*Ultimately a child would use the traditional algorithm that's a shortcut.*" I believe she was completely sincere in her remark and both Carol and Linda nodded agreement. The fact that traditional universal arithmetic algorithms are seen as some sort of "short cuts" by PWCS key leaders speaks volumes. Just for fun, I'd challenge anyone to look up the definition of an "algorithm" in any dictionary or online resource. Nowhere is the word "short cut" used in the description of the term…only MI considers the word something that the rest of the world does not. In any case, the logical follow-on question to Gail's assertion should be, if not by 5th grade (*mindful that other children in traditional/conventional mathematics curricula begin learning the traditional division algorithm in 3rd grade*), just *when* would children reach that "ultimate" goal – traditional division – in the PWCS curriculum?

If not taught at all in MI, is the long division skill "punted" to PWCS middle schools? The Virginia Standards of Learning (SOLs) don't ever specify the *requirement* to provide instruction on traditional algorithms.[2] I think it's fair to say that until recently state Departments of Education haven't had to explicitly state the need for the obvious – it is supposed to be mathematics – one teaches and instills student fluency using the tools of the trade and traditional algorithms are some of the key tools needed for success.[3] The SOLs do however place the proper emphasis on developing proficiency in solving computational arithmetic problems – in the elementary school K-5 standards. By grade 6 the Virginia SOLs have moved on – it's assumed that the basic skills were provided in the previous six years' matriculation in the elementary grades. So just where and when is that "ultimate goal" of long division taught? By Grade 6 the SOLs expect that students "*Will find the quotient, given a dividend expressed as a decimal through thousandths and a divisor expressed as a decimal to thousandths with exactly one non- zero digit.*[4]"

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[2] http://www.doe.virginia.gov/VDOE/Superintendent/Sols/mathsol2001.pdf

[3] http://www.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf

[4] http://www.doe.virginia.gov/VDOE/Superintendent/Sols/mathsol2001.pdf

Not a lot of room to re-explore that which should have been addressed in elementary school. Maybe, just maybe this is why the Virginia Department of Education never approved MI Grade 5 for instruction in our schools. Getting off track a bit; apologies.

The division strategy lesson kind of faded out as the time was dwindling. At the end of the workshop there was time for questions and Linda and Carol offered to stay as long as anyone wanted them to. We had very cordial good byes and rather than filling out the evening's survey at the end I told them I wanted to think it over and would e-mail my comments to them in a day or so. I'm still thinking over what to send.

Overall, it was informative to understand that to Carol and Linda, and apparently the entire PWCS central office staff, Math Investigations is indeed an ideology and philosophy - a total belief system if you will. No matter what instructional/mathematics programmatic direction, if any, is provided from the Board there won't be any balance in the system so long as ideology serves as an acceptable reason to limit mathematical content in the elementary curriculum. I'd have stayed to have a dialogue w/ them; may do so the next time they hold a session, but I wanted to follow the other attending families out and see what they thought. It's really informative to get a feel for what the audience of parents is thinking. I don't think they were buying it, but I think that at least two of the three families attending seemed to be looking at the staff as authority figures and trying to rationalize in their minds that it was going to be OK because the central staff educators said it was "superior math." I did speak with one of the parents in the parking lot as we walked out - mentioned that states are dropping this because of where it leaves kids in the end. She seemed concerned - and rightfully so.

A week later and I'm still dumfounded by the one thing that both Linda and Carol emphasized -- that our teachers in PWCS *don't understand and therefore can't teach the universal arithmetic algorithms*. They just "*do what they learned*" (apparently without comprehension) and not only don't understand these, but can't apparently be taught to teach these. It begs the question of course just how then our teachers can be "trained/professionally developed" to understand and teach open array partial product area model multiplication and relate it to Algebra I operations if they don't understand and can't be trained to teach children simple algorithms that have been within the art of the possible for hundreds of millions of children for going on a couple thousand years.

I'm looking forward to a PWCS workshop on fractions - another math subject that MI just doesn't "*do*" well. MI's content is limited to fractions with sums of less than 2 and there is very limited work with mixed numbers, leaving students unprepared to deal with fractions as simple as 11/3. Notably, common denominators are not well-developed, leaving students ill-prepared to add arbitrary fractions with what they are taught. Guess that's likely another one of those "*ultimate goal short cuts*" that are to be taught at home.

This was a bit longer than intended. Hope you find it informative.

Regards from Prince William County, VA
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