APPENDIX 3

MEASUREMENTS AND SIGNIFICANT FIGURES

EXPRESSING MEASUREMENTS

If you are counting the number of magnets in a box, you will arrive at an *exact* answer—the same answer that anyone else would get unless one of you had made a mistake in counting. On the other hand, if you are measuring the volume of the box, you can only *approximate* the true volume. You might measure the sides and calculate the volume from the measurements. Or you might fill the box with sand and then measure the volume of the sand with a graduated cylinder. No matter what method you use, your measuring tools and techniques will limit your answer to an approximation.

It is very likely that several independent measurements of the volume of the box will give slightly differing results. There are several reasons: the sand may not be perfectly level in the graduated cylinder each time; some sand grains may remain inside the container when it is emptied; the sand may pack down or spread out as it is handled, so its volume may change. No matter how carefully you work, you are not likely to determine the volume exactly. In fact, even if you *had* obtained the result 461.7 ml on each of three trials, you would still know the volume to only a tenth of a milliliter—with the same graduated cylinder there would be no way you could tell whether the volume was exactly 461.700000 . . . ml or whether it was 461.6821 . . . ml or just what it was.

Scientific measurements should be written down in a way that makes clear the amount of certainty in the measurement. Suppose that the results of three trials of measuring the volume of the box gave 461.7 ml, 461.9 ml, and 461.8 ml. The best value for the volume would be the average of the three, or 461.8 ml. Note that this value has three reasonably certain digits—4, 6, and 1—and one rather uncertain digit—8. Someone reading your value would know that, as far as you could tell, the volume was close to 461.8—it could well be a few tenths of a milliliter higher or lower. In general, the results of a measurement should be written with as many digits as are certain, plus one more that is uncertain. When a measurement is written according to this convention, all the certain digits plus the one uncertain digit are called significant figures. It may be necessary to add additional zeroes to put the decimal point in the right position, but this does not alter the number of significant figures (see Counting Significant Figures in following discussion).

If you needed to measure the volume more *precisely*—that is, to more decimal places—you would need different measuring tools or perhaps even a different technique. A value for the volume would be truly more precise only if it had just one uncertain digit in it. For example, suppose three measuring trials gave results of 461.82 ml, 461.84 ml, and 461.80 ml. Then you would be justified in giving the value 461.82 ml as the volume to two decimal places. But if the three trials gave 461.82 ml, 461.95 ml, and 461.69 ml, there would be uncertainty in *both* digits to the right of the decimal point. Even though the average of these three values is 461.82 ml, your data do not justify five significant figures; you would have to give the value as 461.8 ml, which is no more precise than the previous value.

COUNTING SIGNIFICANT FIGURES

The rules for counting significant figures are straightforward.

1. All non-zero digits are significant regardless of the position of the decimal point. Each of the following measurements has five significant figures.

564.32 grams　　87.593 meters　　4531.6 cubic centimeters

2. All zeros between two significant figures are significant. Each of the following measurements has four significant figures.

7.051 meters　　80.05 grams　　100.6 miles

3. When there is no decimal point, zeros at the end of a number are not significant figures. The measurement 42,600 meters has three significant figures (the 4, 2, and 6).

4. If the zero at the end of a number with no decimal point is really the result of a measurement and is meant to be a significant figure, it is often written with a dash above it.

Measurement	Number of significant figures
60,000 grams	1
$6\bar{0},000$ grams	2
$60,\bar{0}00$ grams	3
$60,0\bar{0}0$ grams	4
$60,00\bar{0}$ grams	5

5. When there is a decimal point, zeroes at the end of a number, to the right of the decimal point, are significant figures. The measurement 42.600 meters has five significant figures.

6. Zeros at the beginning of a number are never significant figures. The following measurements all have three significant figures.

0.318 centimeter　　0.0000318 meter　　0.0300 meter

WORKING WITH SIGNIFICANT FIGURES

When you add, subtract, multiply, or divide with significant figures, you must take care that your result does not have more than one uncertain digit. In multiplication and division, the result can have no more significant figures than the measurement with the fewest significant figures in the original data. For example, when 3.112 m is multiplied by 2.2 m, the product 6.8464 m^2 should be rounded off to 6.8 m^2 to avoid more than one uncertain digit. In addition and subtraction, the result can have no more decimal places than the measurement with the fewest decimal places in the original data. For example, when 96.3 g is added to 8.149 g, the result 104.449 g should be rounded off to 104.4 g.

Doing multiplications and divisions with pencil and paper is no more exact than doing them on a slide rule when your final answer has three or fewer significant figures. The slide rule is capable of handling numbers with up to three significant figures. You will rarely make measurements in this course that have more than three. Using a slide rule takes the drudgery out of multiplication and division and gives results that are just as exact as those you would get by doing calculations the long way.