Final Examination-June 2004
AB Calculus

Stuyvesant High School
Department of Mathematics

Directions: Answer all multiple choice questions on the NCS form provided (60 points). You must do all of Part II (14points). In Part III, select two out of the four questions (26 points). Show all work in Part II and Part III.

No Calculators Are Allowed.

PART I

1. Evaluate: $\lim_{x\to -5} \frac{x^2+2x-15}{x+5} =$ a) $-\infty$ b) 2 c) 0 d) ∞ e) -8

2. Evaluate: $\lim_{x\to 8\pi} \sin(x+6\sin x) =$
   a) ∞ b) -1 c) 1 d) 0 e) 8π

3. For what value of the constant c is the function f continuous on $(-\infty,\infty)$?
   $f(x)\begin{cases} cx+7, for\ x\le 2 \\ cx^2-5, for\ x>2 \end{cases}$ a) $c=1$ b) $c=2$ c) $c=6$ d) $c=-2$ e) $c=7$

4. Find an equation of the tangent line to the curve $y=x^3-5x+3$ at the point (2, 1).
   a) $y = 8x + 13$ b) $y = -9x - 13$ c) $y = 7x - 13$ d) $y = -7x + 13$ e) $y = 7x - 15$

5. If a ball is thrown into the air with a velocity of 58 ft/s, its height (in feet) after t seconds is given by $H=58t-11t^2$. Find the velocity when $t = 4$.
   a) -27 ft/s b) -30 ft/s c) 31 ft/s d) 25 ft/s e) 37 ft/s

6. Given $h(x)=\frac{x+2}{x-4}$, then $h'(x)=$
   a) $-\frac{2}{(x-4)^2}$ b) $-\frac{6}{(x-4)^2}$ c) $\frac{2}{(x-4)^2}$ d) $\frac{6}{(x-4)^2}$ e) none of thes

7. Find the inflection points of the graph of $f(x)=8x+2-\sin x, 0<x<3\pi$
   a) $(\pi, 8\pi), (2\pi, 16\pi+2)$ b) $(\pi, 2), (2\pi, 16\pi+2)$ c) $(\pi, 8\pi), (2\pi, 16\pi)$
   d) $(\pi, 8\pi+2), (2\pi, 16\pi+2)$ e) $(\pi, 8\pi+2), (2\pi, 16\pi)$

8. Find the exact values of the numbers c that satisfy the conclusion of The Mean Value Theorem for the function $f(x)=x^3-5x$ for the interval $[-5,5]$.
   a) $c=\frac{5\sqrt{3}}{3}$ b) $c=-\frac{5\sqrt{3}}{3}$ c) $c=\pm\frac{5\sqrt{3}}{3}$ d) $c=\pm 5\sqrt{3}$ e) none of these

9. If $f(x)=\frac{x}{\ln x}$, find $f'(e^4) =$ a) $-\frac{3}{16}$ b) $-\frac{3}{4}$ c) $\frac{3}{4}$ d) $\frac{3}{16}$ e) $-\frac{1}{2}$

10. Differentiate the function $y=x^{e^{7x}}$
   a) $y'=x^{e^{7x}}(7\ln x+\frac{1}{x})$ b) $y'=e^{7x}x^{e^{7x}}(7\ln x+\frac{1}{x^2})$ c) $y'=e^{7x}x^{e^{7x}}(7\ln x-\frac{1}{x})$
   d) $y'=e^{7x}x^{e^{7x}}(7\ln x+\frac{1}{x})$ e) $y'=e^{7x}x^{e^{7x}}(7\ln x-\frac{1}{x^2})$

11. Differentiate the function $y=\frac{e^x+e^{-x}}{e^x-e^{-x}}$
   a) $y'=-\frac{4}{(e^x-e^{-x})^2}$ b) $y'=\frac{4}{(e^x+e^{-x})^2}$ c) $y'=-2\frac{e^{2x}+e^{-2x}}{(e^x-e^{-x})^2}$
   d) $y'=-\frac{2}{e^{2x}-e^{-2x}}$ e) $y'=-\frac{1}{(e^x-e^{-2x})}$

12. Use the TRAPEZOIDAL RULE with n = 3 to approximate the area under $y=x^2$ from $x=1$ to $x=4$.
   a) $\frac{45}{3}$ b) $\frac{43}{3}$ c) $\frac{43}{2}$ d) 43 e) 21

13. Evaluate the integral $\int_0^3 (6+6y-y^2)dy =$
   a) -18 b) 45 c) 54 d) 36 e) -12

14. The acceleration function (in m/s²) and the initial velocity are given for a particle moving along a line. Find the distance traveled for the time interval: $0\le t\le 10$. $a(t)=t+4$, $v(0)=3$
   a) $396\frac{2}{3}m$ b) $391\frac{2}{3}m$ c) $411\frac{2}{3}m$ d) $406\frac{2}{3}m$ e) $415\frac{2}{3}m$

15. Evaluate the integral $\int (x^3 + 2 + \frac{1}{x^2+1})dx$

a) $\frac{x^4}{4} + 2x + \tan^{-1} x + C$ b) $x^4 + 2 + \tan^{-1} x + C$ c) $\frac{x^4}{4} + 2x + \frac{3}{x^3+3} + C$

d) $\frac{x^4}{4} + 2x + \tan^{-1} 2x^2 + C$ e) $4 + 2x + \tan^{-1} x + C$

16. Evaluate the indefinite integral $\int \frac{e^x}{e^x+1} dx =$ a) $-\frac{1}{2}\ln(e^x+1)+C$ b) $\frac{1}{2}\ln(e^x+1)+C$

c) $\ln(e^x-1)+C$ d) $\ln(e^x+1)+C$ e) $-\ln(e^x+1)+C$

17. Evaluate the indefinite integral: $\int x^2\sqrt{x^3+9}dx$

a) $\frac{1}{9}(x^3+9)^{1/2}+C$ b) $\frac{2}{9}(x^3+9)^{3/2}+C$ c) $\frac{2}{9}(x^3-9)^{3/2}+C$

d) $-\frac{2}{9}(x^3+9)^{3/2}+C$ e) $\frac{2}{9}(x^3+9)^{1/2}+C$

18. Evaluate the indefinite integral $\int t^2 \cos(1-t^3)dt$

a) $-\frac{1}{3}\cos(1-t^3)+C$ b) $-\sin(1-t^3)+C$ c) $-\frac{1}{3}\sin(1-t^3)+C$ d) $\frac{1}{3}\sin(1-t^3)+C$ e) $\frac{1}{3}\sin(1-t^2)+C$

19. Find the volume of the solid obtained by rotating the region in the first quadrant bounded by $y = x^2$ and $y = 9$ about the y axis.

a) $9\pi/2$ b) $81/2$ c) $81\pi/2$ d) $81\pi/4$ e) 9π

20. Evaluate the definite integral $\int_{e^{25}}^{e^{64}} \frac{dx}{x\sqrt{\ln x}}$. a) 3 b) 6 c) 39 d) 78 e) none of these

Part II (14 points)
All Students Must Complete this Problem

1. Let R be the region in the first quadrant that is enclosed by the graph $y = \tan x$, the x-axis and the line $x = \frac{\pi}{3}$

   a) Find the area of R
   b) Find the volume of the solid formed by revolving R about the x-axis.

Part III

Select two out of the four given questions. **Show all work**. These problems are worth 13 points each.

1. A particle, initially at rest, moves along the x-axis so that its acceleration at any time $t \geq 0$ is given by $a(t) = 12t^2 -$
   The position of the particle when $t = 1$ is $x(1) = 3$.
   a) Find the values of t for which the particle is at rest.
   b) Write an expression for the position $x(t)$ of the particle at any time $t \geq 0$.
   c) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

2. Given the function f defined by $f(x) = \cos x - \cos^2 x$ for $-\pi \leq x \leq \pi$
   a) Find the x-intercepts of the graph of f
   b) Find the x- and y-coordinates of all relative maximum points of f. Justify your answer.
   c) Find the intervals on which the graph of f is increasing.
   d) Using the information found in parts (a), (b), and (c), sketch the graph of f.

   a) Find a solution of the differential equation $\frac{dy}{dx} = 5x^4y$ that satisfies the initial condition of $y(0) = 2$.