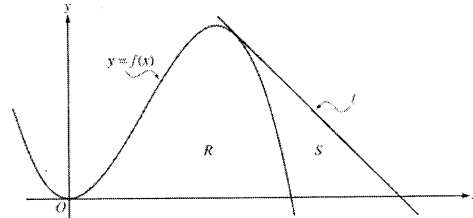


**AP[®] CALCULUS AB
2003 SCORING GUIDELINES (Form B)**

Question 1

Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line $y = 18 - 3x$, where ℓ is tangent to the graph of f . Let R be the region bounded by the graph of f and the x -axis, and let S be the region bounded by the graph of f , the line ℓ , and the x -axis, as shown above.



- Show that ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$.
- Find the area of S .
- Find the volume of the solid generated when R is revolved about the x -axis.

Question 2

A tank contains 125 gallons of heating oil at time $t = 0$. During the time interval $0 \leq t \leq 12$ hours, heating oil is pumped into the tank at the rate

$$H(t) = 2 + \frac{10}{(1 + \ln(t+1))} \text{ gallons per hour.}$$

During the same time interval, heating oil is removed from the tank at the rate

$$R(t) = 12 \sin\left(\frac{t^2}{47}\right) \text{ gallons per hour.}$$

- How many gallons of heating oil are pumped into the tank during the time interval $0 \leq t \leq 12$ hours?
- Is the level of heating oil in the tank rising or falling at time $t = 6$ hours? Give a reason for your answer.
- How many gallons of heating oil are in the tank at time $t = 12$ hours?
- At what time t , for $0 \leq t \leq 12$, is the volume of heating oil in the tank the least? Show the analysis that leads to your conclusion.

Question 3

A blood vessel is 360 millimeters (mm) long with circular cross sections of varying diameter. The table above gives the measurements of the diameter of the blood vessel at selected points

| | | | | | | | |
|----------------------|----|----|-----|-----|-----|-----|-----|
| Distance x (mm) | 0 | 60 | 120 | 180 | 240 | 300 | 360 |
| Diameter $B(x)$ (mm) | 24 | 30 | 28 | 30 | 26 | 24 | 26 |

along the length of the blood vessel, where x represents the distance from one end of the blood vessel and $B(x)$ is a twice-differentiable function that represents the diameter at that point.

- Write an integral expression in terms of $B(x)$ that represents the average radius, in mm, of the blood vessel between $x = 0$ and $x = 360$.
- Approximate the value of your answer from part (a) using the data from the table and a midpoint Riemann sum with three subintervals of equal length. Show the computations that lead to your answer.
- Using correct units, explain the meaning of $\pi \int_{125}^{275} \left(\frac{B(x)}{2}\right)^2 dx$ in terms of the blood vessel.
- Explain why there must be at least one value x , for $0 < x < 360$, such that $B''(x) = 0$.

AP[®] CALCULUS AB
2003 SCORING GUIDELINES (Form B)

Question 4

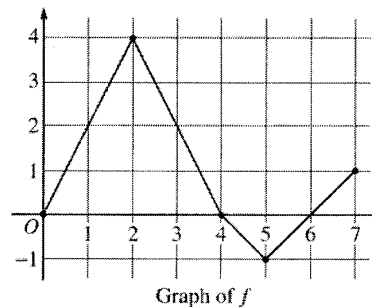
A particle moves along the x -axis with velocity at time $t \geq 0$ given by $v(t) = -1 + e^{1-t}$.

- (a) Find the acceleration of the particle at time $t = 3$.
 - (b) Is the speed of the particle increasing at time $t = 3$? Give a reason for your answer.
 - (c) Find all values of t at which the particle changes direction. Justify your answer.
 - (d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.
-

Question 5

Let f be a function defined on the closed interval $[0, 7]$. The graph of f , consisting of four line segments, is shown above. Let g be the function given by $g(x) = \int_2^x f(t) dt$.

- (a) Find $g(3)$, $g'(3)$, and $g''(3)$.
- (b) Find the average rate of change of g on the interval $0 \leq x \leq 3$.
- (c) For how many values c , where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b)? Explain your reasoning.
- (d) Find the x -coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer.



Question 6

Let f be the function satisfying $f'(x) = x\sqrt{f(x)}$ for all real numbers x , where $f(3) = 25$.

- (a) Find $f''(3)$.
- (b) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = x\sqrt{y}$ with the initial condition $f(3) = 25$.