

**AP<sup>®</sup> CALCULUS AB**  
**2004 SCORING GUIDELINES (Form B)**

**Question 1**

Let  $R$  be the region enclosed by the graph of  $y = \sqrt{x-1}$ , the vertical line  $x = 10$ , and the  $x$ -axis.

- (a) Find the area of  $R$ .
- (b) Find the volume of the solid generated when  $R$  is revolved about the horizontal line  $y = 3$ .
- (c) Find the volume of the solid generated when  $R$  is revolved about the vertical line  $x = 10$ .

**Question 2**

For  $0 \leq t \leq 31$ , the rate of change of the number of mosquitoes on Tropical Island at time  $t$  days is modeled by  $R(t) = 5\sqrt{t} \cos\left(\frac{t}{5}\right)$  mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time  $t = 0$ .

- (a) Show that the number of mosquitoes is increasing at time  $t = 6$ .
- (b) At time  $t = 6$ , is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.
- (c) According to the model, how many mosquitoes will be on the island at time  $t = 31$ ? Round your answer to the nearest whole number.
- (d) To the nearest whole number, what is the maximum number of mosquitoes for  $0 \leq t \leq 31$ ? Show the analysis that leads to your conclusion.

**Question 3**

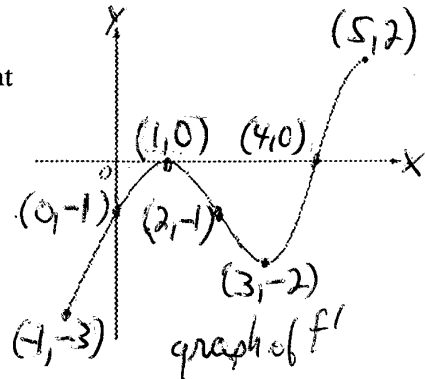
A test plane flies in a straight line with positive velocity  $v(t)$ , in miles per minute at time  $t$  minutes, where  $v$  is a differentiable function of  $t$ . Selected values of  $v(t)$  for  $0 \leq t \leq 40$  are shown in the table above.

$t$ (min)	0	5	10	15	20	25	30	35	40
$v(t)$ (mpm)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

- (a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate  $\int_0^{40} v(t) dt$ . Show the computations that lead to your answer. Using correct units, explain the meaning of  $\int_0^{40} v(t) dt$  in terms of the plane's flight.
- (b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval  $0 < t < 40$ ? Justify your answer.
- (c) The function  $f$ , defined by  $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$ , is used to model the velocity of the plane, in miles per minute, for  $0 \leq t \leq 40$ . According to this model, what is the acceleration of the plane at  $t = 23$ ? Indicate units of measure.
- (d) According to the model  $f$ , given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval  $0 \leq t \leq 40$ ?

### Question 4

The figure above shows the graph of  $f'$ , the derivative of the function  $f$ , on the closed interval  $-1 \leq x \leq 5$ . The graph of  $f'$  has horizontal tangent lines at  $x = 1$  and  $x = 3$ . The function  $f$  is twice differentiable with  $f(2) = 6$ .

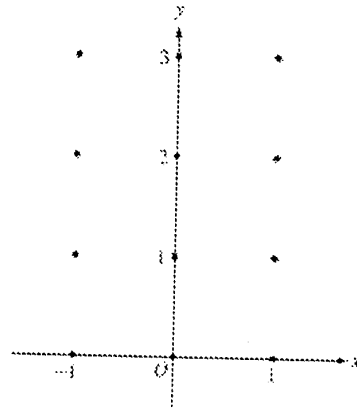


- Find the  $x$ -coordinate of each of the points of inflection of the graph of  $f$ . Give a reason for your answer.
- At what value of  $x$  does  $f$  attain its absolute minimum value on the closed interval  $-1 \leq x \leq 5$ ? At what value of  $x$  does  $f$  attain its absolute maximum value on the closed interval  $-1 \leq x \leq 5$ ? Show the analysis that leads to your answers.
- Let  $g$  be the function defined by  $g(x) = xf(x)$ . Find an equation for the line tangent to the graph of  $g$  at  $x = 2$ .

### Question 5

Consider the differential equation  $\frac{dy}{dx} = x^4(y - 2)$ .

- On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.  
(Note: Use the axes provided in the test booklet.)
- While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the  $xy$ -plane. Describe all points in the  $xy$ -plane for which the slopes are negative.
- Find the particular solution  $y = f(x)$  to the given differential equation with the initial condition  $f(0) = 0$ .



### Question 6

Let  $\ell$  be the line tangent to the graph of  $y = x^n$  at the point  $(1, 1)$ , where  $n > 1$ , as shown above.

- Find  $\int_0^1 x^n dx$  in terms of  $n$ .
- Let  $T$  be the triangular region bounded by  $\ell$ , the  $x$ -axis, and the line  $x = 1$ . Show that the area of  $T$  is  $\frac{1}{2n}$ .
- Let  $S$  be the region bounded by the graph of  $y = x^n$ , the line  $\ell$ , and the  $x$ -axis. Express the area of  $S$  in terms of  $n$  and determine the value of  $n$  that maximizes the area of  $S$ .

