Stuyvesant High School — January 2008
Mr. S. Teitel, Principal — Ms. M. Ferrara, Coordinator

MC1X Final Exam

Calculators are NOT permitted

Part I: Select the best answer for all questions. Record all answers on the NCS form provided.
Part I is worth 60 points.

1. Find $\frac{dy}{dx}$ for $y = 4\sin^2(3x)$.
   (A) 8sin(3x) (B) 24sin(3x) (C) 8sin(3x)cos(3x) (D) 12sin(3x)cos(3x) (E) 24sin(3x)cos(3x)

2. A particle moves along the x-axis so that at any time $t \geq 0$ its position is given by $x(t) = t^3 - 3t^2 - 9t + 1$. For what values of *t* is the particle at rest?
   (A) no values (B) 1 only (C) 3 only (D) 5 only (E) 1 and 3

3. What is the x-coordinate of the point of inflection of the graph of: $y = x^3 + 3x^2 - 45x + 81$?
   (A) −9 (B) −5 (C) −1 (D) 1 (E) 3

4. Suppose $x^2 - xy + y^2 = 3$. Find $\frac{dy}{dx}$ at the point (m,n):
   (A) $\frac{m-2n}{2m-n}$ (B) $\frac{n-2m}{2n-m}$ (C) $\frac{m-2n}{2m+n}$ (D) $\frac{m-2n}{2n+m}$ (E) $\frac{n+2m}{2n+m}$

5. The value of c in the open interval (0,4) that satisfies the Mean Value Theorem for $f(x) = \sqrt{3x+4}$ is:
   (A) 0 (B) $\frac{3}{5}$ (C) $\frac{5}{3}$ (D) 2 (E) 3

6. The side of a cube is expanding at a constant rate of 2 centimeters per second. What is the instantaneous rate of change of the surface area of the cube, in cm^2 per second, when its volume is 27 cubic centimeters?
   (A) 6 (B) 24 (C) 36 (D) 54 (E) 72

7. If p(x) is a continuous function on the closed interval [1,3], with $p(1) \leq K \leq p(3)$ and c is in the closed interval [1,3], then which of the following statements must be true?
   (A) $p(c) = \frac{p(3)+p(1)}{2}$ (B) $p(c) = \frac{p(3)-p(1)}{2}$ (C) There is at least one value c, such that p(c) = K
   (D) There is only one value c, such that p(c) = K (E) c = 2

8. A particle moves in a straight line with velocity $v(t) = 4 - t^2$ feet per second. What is the total distance the particle travels between time $t = 0$ and $t = 3$ seconds? (A) $\frac{7}{3}$ ft (B) 3 ft (C) 6 ft (D) $\frac{23}{3}$ ft (E) $\frac{25}{3}$ ft

9. If $f(x) = -2x^3 + 6x$, then the absolute maximum value of *f(x)* on [−3, 3] is:
   (A) −4 (B) 4 (C) 36 (D) −36 (E) There is no maximum value

10. $\lim_{x\to\infty} \frac{\sqrt{x^2-14}}{3-2x}$ equals: (A) $-\infty$ (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) $\frac{\sqrt{14}}{3}$ (E) ∞

11. The position of an object moving along a straight line for $t \geq 0$ is given by $s_1(t) = t^3 + 2$, and the position of a second object moving along the same line is given by $s_2(t) = t^2$. If both objects begin at $t = 0$, at what time is the distance between the objects a minimum? (A) 2 (B) $\frac{50}{27}$ (C) $\frac{2}{3}$ (D) 0 (E) none of these

12. For $x \neq 0$, the slope of the tangent to $y = x\cos x$ equals zero whenever:
   (A) $\tan x = -x$ (B) $\tan x = \frac{1}{x}$ (C) $\tan x = x$ (D) $\sin x = x$ (E) $\cos x = x$

13. $\int t\cos(2t)^2\,dt =$ (A) $\frac{1}{8}\sin(4t^2) + C$ (B) $\frac{1}{2}\cos^2(2t) + C$ (C) $-\frac{1}{8}\sin(4t^2) + C$
   (D) $\frac{1}{4}\sin(2t)^2 + C$ (E) none of these

14. $\int x\sqrt{x-1}\,dx =$ (A) $\frac{x^2}{2} + \frac{1}{2}(x-1)^{-\frac{1}{2}} + C$ (B) $\frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}} + C$
   (C) $\frac{2}{3}(x-1)^{\frac{3}{2}} + (x-1)^{\frac{1}{2}} + C$ (D) $\frac{x^4}{4}(x-1)^{\frac{3}{2}} + C$ (E) $x^2(x-1)^{-\frac{1}{2}} + C$

15. Determine the value of k so that f(x) is continuous on the entire real line when $f(x) = \begin{cases} x+3, & x \leq -1 \\ 2x-k, & x > -1 \end{cases}$
   (A) −4 (B) −1 (C) 0 (D) 1 (E) 4

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16. Using four equal subintervals, the left sum approximation of $\int_1^5 x^2 dx$ equals:

(A) 30 (B) $41\frac{1}{3}$ (C) 42 (D) 54 (E) 55

17. $\int \frac{\cos x}{\sqrt{1+\sin x}} dx =$ (A) $-\frac{1}{2}(1+\sin x)^{1/2}+C$ (B) $\frac{1}{2}(1+\sin x)^{1/2}+C$ (C) $2\sqrt{1+\sin x}+C$

(D) $-2\sqrt{1+\sin x}+C$ (E) $\frac{2}{3(1+\sin x)^{3/2}}+C$

18. Given that $f(-3)=4$ and $f'(-3)=2$, which of the following is the tangent line approximation of $f(-3.1)$?

(A) 3.8 (B) 3.9 (C) 4.0 (D) 4.1 (E) 4.2

19. $\lim_{x\to 0}\frac{\sin x}{2x}+\lim_{x\to 0}\frac{1-\cos x}{x} =$ (A) 0 (B) $\frac{1}{2}$ (C) $\frac{3}{2}$ (D) 2 (E) undefined

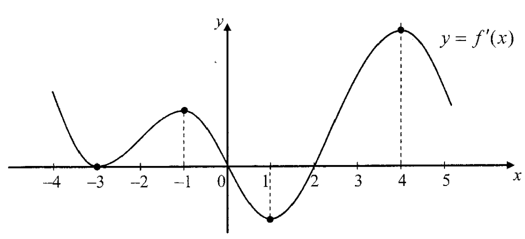
20. $\lim_{x\to 64}\frac{\sqrt{x}-8}{x-64}$ equals: (A) $\frac{1}{16}$ (B) $\frac{1}{24}$ (C) $\frac{1}{32}$ (D) 1 (E) undefined

Part II: You must complete each of the following problems. Write your answers in the booklet provided, and SHOW ALL WORK. Part II is worth 40 points.

1. A closed box with a square base is needed to package 100 cm^3 of powdered milk. Find the dimensions of the box that will use the minimum amount of material. [7 points]

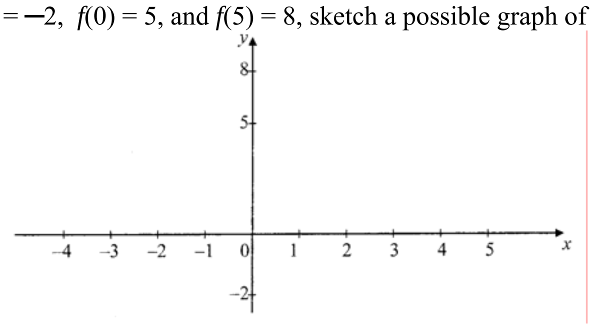
2. The function $f(x)=\frac{x}{1+x^2}$ and its derivative is $f'(x)=\frac{1-x^2}{(1+x^2)^2}$ [11 points]

(a) Describe the symmetry of the graph of *f*.
(b) Write an equation for each vertical and horizontal asymptote, if any.
(c) Find the relative max and min, if any.
(d) Find any points of inflection, if any.
(e) Use the results of (a) – (d) to sketch an accurate graph of *f*.



3. The figure above shows the graph of *f'*, the derivative of the function *f*, for $-4 \le x \le 5$. The graph of f' has horizontal tangent lines at $x = -3, -1, 1$, and 4. [11 points]

(a) Find all the values of *x*, for $-4 < x < 5$, for which *f* is decreasing. Justify your answer.
(b) Find all the values of *x*, for $-4 < x < 5$, at which *f* attains a relative maximum. Justify your answer.
(c) Find all the values of *x*, for $-4 < x < 5$, for which the graph of *f* is concave up.
(d) Given $f(-4) = -2$, $f(0) = 5$, and $f(5) = 8$, sketch a possible graph of *f* copying the axes shown below onto your answer sheet..



4. A balloon rises vertically at a constant rate of 60 feet per minute. A camera is placed 400 feet horizontally from where the balloon was launched. At the moment that the balloon has risen 300 feet, find: [11 points]

(a) The rate at which the distance between the camera and the balloon is changing.
(b) The rate at which the camera's angle of elevation is changing to keep the balloon in the picture.