Stuyvesant High School — January 2007
S. Teitel, Principal — M. Ferrara, Coordinator

MAXX Final Exam

Calculators are NOT permitted.

PART I: Select the *best* answer for all questions. Record all answers on the NCS form provided.
Part 1 is worth 60 points.

1. If $y=\dfrac{3}{4+x^2}$, then $\dfrac{dy}{dx}$ equals: (A) $\dfrac{3}{2x}$ (B) $\dfrac{3x}{(1+x^2)^2}$ (C) $\dfrac{6x}{(4+x^2)^2}$ (D) $\dfrac{-6x}{(4+x^2)^2}$ (E) $\dfrac{-3}{(4+x^2)^2}$

2. $\lim\limits_{h\to 0}\dfrac{8\left(\frac{1}{2}+h\right)^5-8\left(\frac{1}{2}\right)^5}{h}$ = (A) 0 (B) $5/8$ (C) $5/4$ (D) $5/2$ (E) 5

3. If y + cos y = x + 1, $\dfrac{dy}{dx}$ equals:

   (A) $\dfrac{1}{1+\sin y}$ (B) $\dfrac{1}{1-\sin y}$ (C) 1 + sin y (D) 1 − sin y (E) −sin y

4. The Mean Value Theorem guarantees the existence of a special point on the graph of $y=\sqrt{x}$ between $(0,0)$ and $(4,2)$. The coordinates of this point are:

   (A) $(2,1)$ (B) $(1,1)$ (C) $(2,\sqrt{2})$ (D) $\left(\dfrac{1}{2},\dfrac{1}{\sqrt{2}}\right)$ (E) None of these

5. The value of k for which f(x) = $x+\dfrac{k}{x}$ has a relative maximum at $x=-2$ is:

   (A) -4 (B) -2 (C) 2 (D) 4 (E) None of these

6. The graph of $y=5x^4-x^5$ has a point of inflection at:

   (A) $(0,0)$ only (B) $(3,162)$ only (C) $(4,256)$ only (D) $(0,0)$ and $(3,162)$ (E) $(0,0)$ and $(4,256)$

7. $\lim\limits_{x\to -9}\dfrac{x^2+6x-27}{x+9}$ (A) ∞ (B) 0 (C) −3 (D) −9 (E) −12

8. $\dfrac{d}{dx}\left[\dfrac{1+\cos x}{1-\cos x}\right]=$ (A) $\dfrac{-2\sin x}{(1-\cos x)^2}$ (B) $\dfrac{2\sin x}{(1-\cos x)^2}$ (C) $\dfrac{-2\cos x}{(1-\cos x)^2}$ (D) $\dfrac{2\cos x}{(1-\cos x)^2}$ (E) $\dfrac{\sin x}{(1-\cos x)^2}$

9. At t = 0, a particle starts at rest and moves along the x-axis from the initial position of 0 in such a way that at time t its acceleration is $24t^2$ feet per second per second. What is its position after the first 2 seconds?

   (A) 32 (B) 48 (C) 64 (D) 96 (E) 192

10. Using differentials or the tangent line method, an estimate of $\sqrt{102}$ is

    (A) 10.2 (B) 10.1 (C) 10.05 (D) $10\frac{1}{21}$ (E) $10\frac{2}{21}$

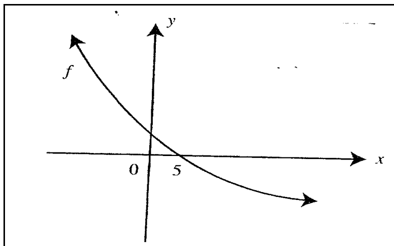
11. If $y=\cos^2 3x$, then $\dfrac{dy}{dx}$ equals:

    (A) $-6\sin 3x\cos 3x$ (B) $-2\cos 3x$ (C) $2\cos 3x$ (D) $6\cos 3x$ (E) $2\sin 3x\cos 3x$

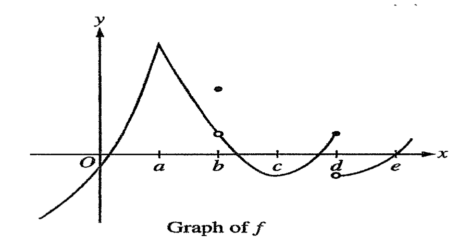
12. If $\begin{cases} f(x) = \dfrac{\sqrt{2x+5} - \sqrt{x+7}}{x-2}, \text{ for } x \neq 2 \\ f(2) = k \end{cases}$ and if f is continuous at $x = 2$, then $k =$

(A) 0 (B) $\dfrac{1}{6}$ (C) $\dfrac{1}{3}$ (D) 1 (E) $\dfrac{7}{5}$

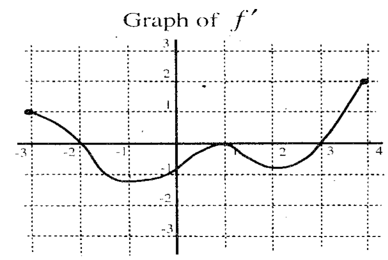
13. A particle starts at time t = 0 and moves along a number line so that its position, at time $t \geq 0$, is given by $x(t) = (t-2)^3(t-6)$. The particle is moving right for: (A) $0 < t < 5$ (B) $2 < t < 6$ (C) $t > 5$ (D) $t \geq 0$ (E) never



For question **14**



For question **15**



For question **16**

14. The graph of function f is shown above *left*. If f is twice differentiable, which of the following statements is true?
(A) $f(5) < f'(5) < f''(5)$ (B) $f''(5) < f'(5) < f(5)$ (C) $f'(5) < f(5) < f''(5)$
(D) $f'(5) < f''(5) < f(5)$ (E) $f''(5) < f(5) < f'(5)$

15. The graph of function f is shown above *center*. The value of x at which f is continuous but not differentiable is:
(A) a (B) b (C) c (D) d (E) e

16. The graph of f', the derivative of function f, is shown above *right*. The domain is the closed interval $[-3, 4]$. Which of the following is true?
I. f is increasing on the interval (2, 4) II. f has a relative minimum at $x = -2$ III. The f-graph has an inflection point at $x = 1$
(A) I only (B) II only (C) III only (D) I and II only (E) I, II, III

17. $\displaystyle\int \frac{\sin x}{\sqrt{1+\cos x}}\,dx =$ (A) $\frac{1}{2}\sqrt{1+\cos x} + C$ (B) $-\frac{1}{2}\sqrt{1+\cos x} + C$ (C) $2\sqrt{1+\cos x} + C$

(D) $-2\sqrt{1+\cos x} + C$ (E) $\sqrt{1+\cos x} + C$

18. $\displaystyle\int \sec x \tan^3 x\,dx =$ (A) $\dfrac{\sec^3 x}{3} - \sec x + C$ (B) $\dfrac{\sec^3 x}{3} + \sec x + C$ (C) $\dfrac{\tan^4 x}{4} + C$

(D) $\dfrac{\tan^4 x}{4} - \dfrac{\tan^2 x}{2} + C$ (E) $\dfrac{\tan^4 x}{4} + \dfrac{\tan^2 x}{2} + C$

19. If $xy + x^2 = 6$, the value of $\dfrac{dy}{dx}$ at $x = -1$ is: (A) -7 (B) -2 (C) 0 (D) 1 (E) 3

20. $f(x) = \dfrac{\sqrt{4x^2-3}}{x+2}$ Which of the following are asymptotes to the graph of f?

I. $y = 2$ II. $y = -2$ III. $y = 0$ IV. $y = 2x$

(A) I (B) II (C) III (D) IV (E) 1, II

PART II: You must complete each of the following problems. Write your work and answers in the booklet provided.

Part II is worth 40 points. (SHOW YOUR WORK)

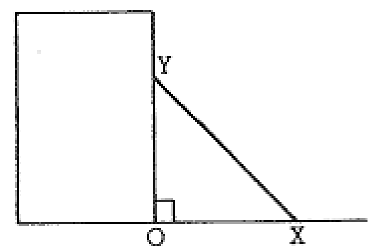
21. $f(x) = 4x^3 - 21x^2 + 36x - 4$. Find the interval or intervals where *f* is decreasing and concave up. [8 points]

22. The function $f(x) = \dfrac{9x^2 - 36}{x^2 - 9}$, and its derivative is $f'(x) = \dfrac{-90x}{(x^2 - 9)^2}$. [12 points]

(a) Describe the symmetry of the graph of *f*.
(b) Write an equation for each horizontal and vertical asymptote of the graph.
(c) Find and state the intervals on which *f* is increasing.
(d) Using the results found in parts (a), (b), and (c), sketch the graph of f.

23. A ladder 15 feet long is leaning against a building so that end X is on level ground and end Y is on the wall as shown in the figure. X is moved away from the building at the constant rate of ½ foot per second. [12 points]

(a) Find the rate in feet per second at which the length OY is changing when X is 9 feet from the building.
(b) Find the rate of change in square feet per second of the area of triangle XOY when X is 9 feet from the building..



24. A rectangle ABCD with sides parallel to the coordinate axes is inscribed in the region enclosed by the graph of $y = -4x^2 + 4$ and the X-axis as shown in the figure.

Find the x and y coordinates of C so that the area of rectangle ABCD is a maximum. [8 points]