Stuyvesant High School
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MAXX FINAL EXAMINATION

Calculators are NOT permitted on this exam.

Part I. Answer all questions. Place all answers on the NCS sheet provided. Part I is worth 60 points.

1. $\lim_{x \to -1} \frac{x^3 + x^2}{x^2 - 1} =$ (A) 2 (B) - 2 (C) ½ (D) - ½ (E) - ∞

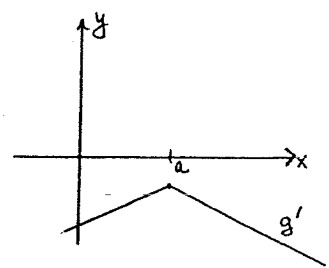
2. $\lim_{x \to 0} \frac{1 - \cos 4x}{x} =$ (A) 0 (B) 1 (C) ¼ (D) 4 (E) ∞

3. Let $f(x) = \begin{cases} \frac{x^2 - 1}{x - 1} & if \quad x \neq 1 \\ 1 & if \quad x = 1 \end{cases}$ Which of the following statements is true?

(A) f(x) is continuous at x = 1 because f(x) is defined at x = 1.
(B) f(x) is continuous at x = 1 because $\lim_{x \to 1} f(x)$ exists.
(C) f(x) is NOT continuous at x = 1 because f (x) is not defined at x = 1.
(D) f(x) is NOT continuous at x = 1 because $\lim_{x \to 1} f(x)$ does not exist.
(E) f(x) is NOT continuous at x = 1 because $\lim_{x \to 1} f(x) \neq f(1)$.

4. $\lim_{h \to 0} \frac{(2+h)^3 - 2^3}{h}$ (A) 36 (B) 12 (C) 8 (D) 2 (E) 0

5. The graph of the derivative of g is shown in the diagram to the right. Which of the following statements are true about g at x = a ?



I. g(x) is continuous
II. g(x) is differentiable
III. g(x) is increasing

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

6. If $f(x) = \sin^2(3x)$, then $\frac{dy}{dx} =$

(A) 2 sin 3x (B) -6 sin 3x cos 3x (C) 6 sin 3x cos 3x (D) 6 sin 3x (E) 2 sin 3x cos 3x

7. $g(x) = \frac{3}{4 + x^2}$, then $g'(1) =$ (A) 3/2 (B) ¾ (C) 6/25 (D) -6/25 (E) -3/25

8. The slope of the tangent to the graph of $4x^2 + 25y^2 = 100$ at the point $P\left(3, \frac{8}{5}\right)$ is

(A) 0 (B) -3/10 (C) 3/10 (D) -3/20 (E) 5/12

9. Let f be defined by $(x - 1)(x + 2)^3$. The function f is increasing on the interval
(A) (- ∞ , - 2) (B) (- 2 , 1) (C) (1 , ∞) (D) (- 2 , ¼) (E) (1/4 , ∞)

10. The graph of $f(x) = x^{\frac{4}{3}} + 4x^{\frac{1}{3}}$ has a point of inflection at
(A) x = 0 only (B) x = 2 only (C) x = 0 and x = 2 (D) x = 0 and x = 1 (E) no point

11. The derivative of $f(x)$ is $f'(x) = \sin(2x)$. The graph of f(x) is concave up on the interval
(A) $\left(0, \frac{\pi}{2}\right)$ (B) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (C) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (D) $\left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$ (E) $\left(\frac{3\pi}{2}, 2\pi\right)$

12. If $f(x) = \frac{x}{x^2 + 1}$, then the graph of f(x) has a relative maximum point at

(A) x = -2 (B) x = -1 (C) x = ½ (D) x = 1 (E) x = 3/2

13. The value of c that satisfies the Mean Value Theorem for $f(x) = 2 + \frac{3}{x}$ on the interval [1 , 6] is

(A) 3.5 (B) $-\frac{1}{2}$ (C) $\sqrt{3}$ (D) $\sqrt{5}$ (E) $\sqrt{6}$

14. The position function of an object moving on a horizontal number line is

$x(t)=\dfrac{t^3}{3}-3t^2+8t+4,\ t\geq 0$. When the acceleration of the object is 0, its velocity is

(A) -1 (B) 0 (C) 1 (D) 3 (E) 8

15. Air is pumped into a spherical balloon at the rate of 16π cm^3/sec. When the radius is 4 cm., the rate of change of the radius is

(A) 1 cm/sec (B) 2 cm/sec (C) $\frac{1}{2}$ cm/sec (D) $\frac{1}{4}$ cm/sec (E) $\sqrt[3]{24}$ cm/sec NOTE: $V=\frac{4}{3}\pi r^3$

16. Using differentials, if the length of a side of a square is decreased from 10 in. to 9.8 in., the approximate change in the area is

(A) - 3.96 in^2 (B) - 4 in^2 (C) - 4.04 in^2 (D) - 0.16 in^2 (E) - 0.04 in^2

17. $\int_0^2 (2x^3+3)dx=$ (A) 8 (B) 11 (C) 14 (D) 20 (E) 24

18. $\int \sec^2\left(\frac{x}{5}\right)dx=$ (A) $5\tan^2\left(\frac{x}{5}\right)+C$ (B) $5\tan\left(\frac{x}{5}\right)+C$

(C) $\tan^2\left(\frac{x}{5}\right)+C$ (D) $\frac{1}{5}\tan^2\left(\frac{x}{5}\right)+C$ (E) $\frac{1}{5}\tan\left(\frac{x}{5}\right)+C$

19. $\int \frac{x}{\sqrt{1-2x^2}}dx=$ (A) $2\sqrt{1-2x^2}+C$ (B) $\frac{1}{2}(1-2x^2)^{\frac{3}{2}}+C$ (C) $-\frac{1}{2}\sqrt{1-2x}+C$

(D) $-\frac{1}{2}\sqrt{1-2x^2}+C$ (E) $\sqrt{1-2x^2}+C$

20. If $f''(x)=6x+6$, $f'(0)=-9$, and $f(0)=-27$ then $f(x)=$

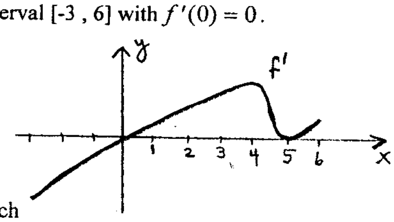
(A) $3x^2+6x-9$ (B) $x^3+3x^2-9x-27$ (C) $x^3+6x-27$
(D) $6x^3+6x^2+x-27$ (E) $6x-30$

Part II. Answer three out of the four questions on the NCS answer sheet provided. Show all work. Part II is worth 40 points.

1. An object moves along a line so that at any time t its velocity is given by $v(t)=3t^2-30t+48$, $t\geq 0$, where t is in seconds and v is in feet per second.

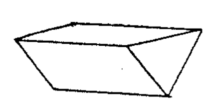
a) Find s(t), the position function for the object, if s(1) = 10.
b) For what values of t does the object move to the right?
c) Find the acceleration of the object at t = 4.
d) Find the minimum velocity of the object for $0\leq t\leq 6$.
e) Find the total distance the object travels for $0\leq t\leq 6$.

2. Given the graph of the derivative function, $y=f'(x)$, on the interval [-3 , 6] with $f'(0)=0$. The graph of $f'(x)$ is tangent to the x-axis at x = 5 and reaches a maximum value at x = 4. Answer each of the following questions about the graph of y = f(x). Provide clear and concise explanations where required.



a) On which interval(s) does f(x) increase ? Justify your answer.
b) Determine each value of x at which f has a relative extrema. In each case, state whether maxima or minima. Justify your conclusion(s).
c) Identify interval(s) in which the graph of f is concave down. Justify your conclusion.

3. A trough is 12 feet long and 3 feet across the top. Its ends are isosceles triangles with altitudes of 3 feet. If water is being pumped into the trough at 2 cubic feet per minute, how fast is the water level rising when the water is 1 foot deep ?



4. The sum of the squares of two positive numbers is 200. Find the maximum value of their product. Your work must include the appropriate derivative test(s) to justify your answer.