Stuyvesant High School
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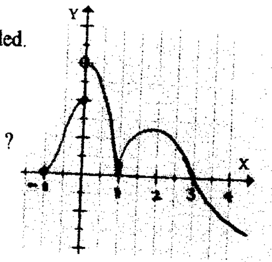
MAXX FINAL EXAMINATION

Calculators are NOT permitted on this exam.

Part I. Answer all questions. Place all answers on the NCS sheet provided.
Part I is worth 60 points.




1. The graph of the function g is shown in the diagram to the right.
For what value(s) of x in the interval $-1 < x < 4$ is g not differentiable ?

(A) 0 only (B) 0 and 1 only (C) 0, 1, and 3 only
(D) 0 and 3 only (E) 1 and 3 only

2. $\lim_{h\to 0}\dfrac{\tan\left(\frac{\pi}{6}+h\right)-\tan\frac{\pi}{6}}{h}$ equals (A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\sqrt{3}$ (D) $\frac{\sqrt{3}}{3}$ (E) $\frac{4}{3}$

3. If $f(x)=\begin{cases} x+k & \text{if } x\le 1 \\ x^2-2x+4 & \text{if } x>1 \end{cases}$, find k such that f(x) is continuous at x = 1

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

4. The slope of the tangent to the graph of $y=\dfrac{x^2}{x^3+3}$ at x = -1 is

(A) $\frac{5}{4}$ (B) $-\frac{5}{4}$ (C) $\frac{7}{4}$ (D) $-\frac{7}{4}$ (E) $-\frac{7}{16}$

5. If the function $f(x) = 2x^3 - 3x^2 - 12x$ is defined over the interval [-3 , 2], find the absolute minimum value of f(x).

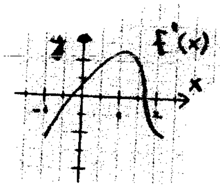
(A) -20 (B) 7 (C) -13 (D) -4 (E) -45

6. Find the derivative dy/dx for the function defined by $y=\frac{1}{2}x^2\sin\left(x^2\right)$

(A) $x^3\cos\left(x^3\right)$ (B) $\frac{1}{2}x^2\cos\left(x^2\right)+x\sin\left(x^2\right)$ (C) $x\sin\left(x^2\right)-x^3\cos\left(x^2\right)$
(D) $x\sin\left(x^2\right)-x^3\sin\left(x^2\right)\cos\left(x^2\right)$ (E) $x\sin\left(x^2\right)+x^3\cos\left(x^2\right)$

7. The critical numbers of $f(x)=x\sqrt{16-x^2}$ are ____________

(A) $\pm 2\sqrt{2}$ (B) $\pm 4, \pm 2\sqrt{2}$ (C) $0,\ \pm 4$ (D) $0,\ \pm 2\sqrt{2}$ (E) $\pm 2, \pm 4$




8. The diagram to the right shows the graph of $f'(x)$, the derivative of $f(x)$. In the interval from $0 < x < 1$, the function $f(x)$ is
(A) increasing and concave up (B) increasing and concave down
(C) decreasing and concave up (D) decreasing and concave down
(E) cannot be determined

9. A tank is being filled with water at a rate of $300\sqrt{t}$ gallons per hour (with $t > 0$ measured in hours). If the tank originally contained 100 gallons, how many gallons of water are in the tank after 4 hours ?
(A) 700 (B) 1000 (C) 1300 (D) 1700 (E) 2500

10. The value(s) of c that satisfies the conditions of the Mean Value Theorem for the function $f(x)=x^3-3x+1$ on the interval [0 , 3] is

(A) 9 (B) $\sqrt{3}$ (C) 1 (D) 2 (E) $\frac{1}{2}$

11. A 20 foot ladder is leaning against the vertical wall of a building. It begins to slide down the wall at a rate of 0.5 ft/sec. At what rate is the base of the ladder moving away from the wall when the base is at a distance of 12 feet from the wall ?
(A) 0.5 ft/sec (B) 2/3 ft/sec (C) 4/3 ft/sec (D) 5/8 ft/sec (E) 8/3 ft/sec

12. The function g(x) = x cos x, defined on the interval $0 < x < \pi$, has an inflection whenever we find that

(A) tan x = x/2 (B) tan x = 2/x (C) tan x = -x/2 (D) tan x = x (E) sin x = x

13. Given the implicit relation 2y - x cos y + xy = 1, evaluate dy/dx when y = 0

(A) 1/3 (B) -1 (C) 2 (D) 1 (E) -2

14. If $f''(x)=4$, $f'(0)=2$, and $f(1)=1$ then $f(x)$ equals

(A) $2x^2+2x$ (B) $4x^2+2x-3$ (C) $2x^2+2x-3$ (D) $2x^2+2x+3$ (E) $4x^2-2x$

15. Evaluate the following integral: $\int x\sqrt{1+\sin x^2}\cos x^2 dx$

(A) $\frac{2}{3}\left(1+\sin x^2\right)^{\frac{3}{2}}+C$ (B) $\frac{1}{2}\left(1+\sin x^2\right)+C$ (C) $-\frac{1}{3}\sin x^2\left(1+\sin x^2\right)^{\frac{3}{2}}+C$

(D) $\frac{1}{3}\left(1+\sin x^2\right)^{\frac{3}{2}}+C$ (E) $\frac{2}{3}x\sqrt{1+\sin x^2}+C$

16. $\int \cos 5x\, dx =$ (A) $-\frac{1}{5}\sin 5x+c$ (B) $\frac{1}{5}\sin 5x+c$ (C) - sin 5x + c

(D) sin 5x + c (E) 5 cos 5x + c

17. $\lim_{x\to-\infty}\frac{2x+1}{\sqrt{x^2-3}}$ (A) - 2 (B) $-\frac{1}{3}$ (C) $-\frac{2}{3}\sqrt{3}$ (D) 1 (E) 2

18. A particle moves in a straight vertical line such that its position function is given by $s(t) = (t-3)(t-7)^3$, $t \geq 0$. At what time is the particle moving down ?

(A) $t > 4$ (B) $4 < t < 7$ (C) $3 < t < 7$ (D) $t > 7$ (E) $0 \leq t < 4$

19. Use the differential to find an approximation for $\sqrt[4]{16.8}$

(A) 2 (B) $\frac{81}{40}$ (C) $\frac{84}{40}$ (D) $\frac{65}{32}$ (E) $\frac{69}{32}$

20. In which interval(s) is the function $f(x)=\frac{2(x^2-9)}{x^2-4}$ increasing ?

(A) $x<-2, -2<x<0$ (B) $x\neq 0, x\neq\pm 2$ (C) $0<x<2, x>2$

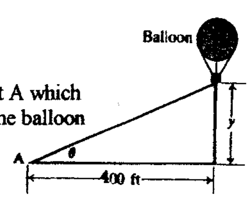
(D) $-3<x<3$ (E) $x<-2$ or $x>2$

**Part II.** (16 points) Analyze and carefully sketch the graph of $f(x)=12x^{\frac{2}{3}}-4x$. Your analysis must include the application of derivative test(s) to verify relative extrema and intervals of concavity. In your diagram, label all intercepts, relative extrema, and points of inflection.

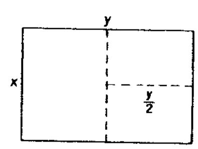
**Part III**. Answer two (2) questions. Each question is worth 12 points.

1. A ball is rolled across a level floor with an initial velocity of 10 feet per second. If the speed of the ball is decreasing at a rate of 3 ft/sec $^2$, how far will the ball roll ?

2. A balloon rising vertically from the ground at 120 ft/min is tracked from a station at point A which is 400 feet from the point of liftoff. Find the rate at which the angle of elevation from A to the balloon is changing when the balloon is 400 feet above the ground.




3. Consider the floor plan in the diagram to the right. The total area is to be 750 square feet, and exterior walls cost twice as many dollars per foot as the interior walls. What dimensions should be chosen to minimize the cost of the walls ?




4. Find the equation of a tangent line to the graph of f(x) = cos x that can be used to approximate the value of $\cos\left(\frac{\pi}{6}+0.1\right)$. Then, find an approximation of $\cos\left(\frac{\pi}{6}+0.1\right)$.