Stuyvesant High School
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MAXX FINAL EXAMINATION

Calculators are NOT permitted on this exam.

PART I. Select the *best* answer for all questions. Place all answers on the NCS sheet provided.
Part 1 is worth 60 points.

1. Let *f* be the function defined by $f(x) = 4x^3 - 5x + 3$. Which of the following is an equation of the line tangent to the graph of *f* at the point where x = –1?
(A) y = 7x – 3 (B) y = 7x + 7 (C) y = 7x + 11 (D) y = –5x – 1 (E) y = –5x – 5

2. $\lim_{x\to\infty} \dfrac{x^3 - 2x^2 + 3x - 4}{4x^3 - 3x^2 + 2x - 1} =$ (A) 4 (B) 1 (C) ¼ (D) 0 (E) –1

3. $\lim_{\theta\to 0} \dfrac{1-\cos\theta}{2\sin^2\theta} =$ (A) 0 (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) 1 (E) nonexistent

4. $f(x) = \begin{cases} x+2, & x \le 3 \\ 4x-7, & x > 3 \end{cases}$ Which of the following statements is true about function *f*?
I. $\lim_{x\to 3} f(x)$ exists II. *f* is continuous at x = 3 III. *f* is differentiable at x = 3
(A) None (B) I only (C) II only (D) I and II only (E) I, II, and III

5. If $f(x) = \sin\dfrac{x}{2}$, then there exists a number *c* in the interval $\dfrac{\pi}{2} < x < \dfrac{3\pi}{2}$ that satisfies the conclusion for the Mean Value Theorem. Which of the following could be *c*? (A) $\frac{2\pi}{3}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{6}$ (D) π (E) $\frac{3\pi}{2}$

6. The function *f* is continuous for –2 ≤ x ≤ 1 and differentiable for –2 < x < 1 . If f(–2) = –5 and f(1) = 4, which of the following statements must be true?
(A) There exists *c*, where –2 < c < 1, such that f(c) = 6. (B) There exists *c*, where –2 < c < 1, such that `f'(c)` = 0.
(C) There exists *c*, where –2 < c < 1, such that f(c) = –6. (D) There exists *c*, where –2 < c < 1, such that `f'(c) = 3`.
(E) There exists *c*, where –2 < c < 1, such that f(c) ≥ f(x) for all x on the interval –2 ≤ c ≤ 1.

7. If $y = x^2\sin 2x$, then $\dfrac{dy}{dx} =$
(A) 2x cos 2x (B) 4x cos 2x (C) 2x(sin 2x + cos 2x)
(D) 2x(sin 2x – x cos 2x) (E) 2x(sin 2x + x cos 2x)

8. If $y = \dfrac{2x+3}{3x+2}$, then $\dfrac{dy}{dx} =$ (A) $\dfrac{12x+13}{(3x+2)^2}$ (B) $\dfrac{12x-13}{(3x+2)^2}$ (C) $\dfrac{5}{(3x+2)^2}$ (D) $\dfrac{-5}{(3x+2)^2}$ (E) $\frac{2}{3}$

9. If $f(x) = (x^2 - 2x - 1)^{\frac{2}{3}}$, then `f'(0)` is (A) $\frac{4}{3}$ (B) 0 (C) $-\frac{2}{3}$ (D) $-\frac{4}{3}$ (E) -2

10. A particle moves along the x-axis so that at time t ≥ 0 its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time *t* is the particle at rest?
(A) t = 1 only (B) t = 3 only (C) t = $\frac{7}{2}$ only (D) t = 3 or t = $\frac{7}{2}$ (E) t = 3 or t = 4

11. What is the slope of the line tangent to the curve $3y^2 - 2x^2 = 6 - 2xy$ at the point (3, 2)?
(A) 0 (B) $\frac{4}{9}$ (C) $\frac{7}{9}$ (D) $\frac{6}{7}$ (E) $\frac{5}{3}$

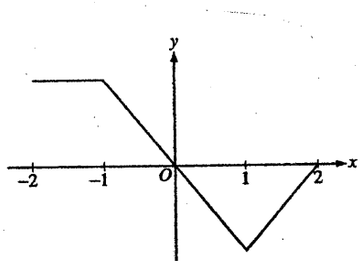
12. The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase of the area of the circle at the instant when the circumference of the circle is 20π meters?
(A) 0.04π m^2/sec (B) 0.4π m^2/sec (C) 4π m^2/sec (D) 20π m^2/sec (E) 100π m^2/sec

13. The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?
(A) $-\frac{7}{8}$ feet/min (B) $-\frac{7}{24}$ feet/min (C) $\frac{7}{24}$ feet/min (D) $\frac{7}{8}$ feet/min (E) $\frac{21}{25}$ feet/min

14. At what value of x does the graph of $y = \dfrac{1}{x^2} - \dfrac{1}{x^3}$ have a point of inflection?
(A) 0 (B) 1 (C) 2 (D) 3 (E) At no value of x

15. The graph of f', the derivative of f, is shown to the right. Which of the following statements is true about f?

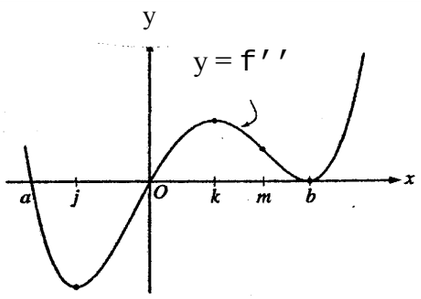
(A) f is decreasing for $-1 \le x \le 1$
(B) f is increasing for $-2 \le x < 0$
(C) f is increasing for $1 \le x \le 2$
(D) f has a local minimum at x = 0
(E) f is not differentiable at x = 1 and x = –1



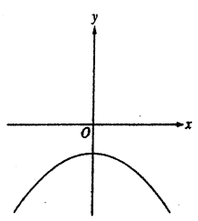
Graph of f'

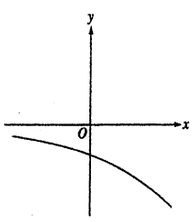
16. The second derivative of the function f is given by $f''(x) = x(x-a)(x-b)^2$. The graph of f'' is shown to the right. For what values of x does the graph of f have an inflection point?

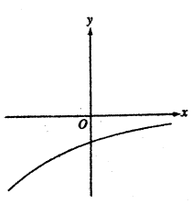
(A) 0 and a only
(B) 0 and m only
(C) b and j only
(D) 0, a, and b
(E) b, j, and k

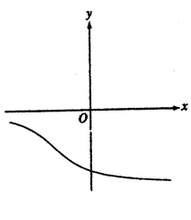


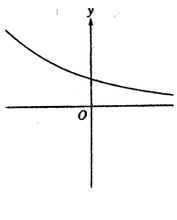
17. The function f has the property that f(x), f'(x), and f''(x) are negative for all real values of x. Which of the following could be the graph of f ?

(A) 

(B) 

(C) 

(D) 

(E) 

18. Consider all the right circular cylinders for which the sum of the height and the circumference is 30 centimeters. What is the radius of the one with the maximum volume?

(A) 3 cm (B) 10 cm (C) 20 cm (D) $\frac{30}{\pi^2}$ cm (E) $\frac{10}{\pi}$ cm

19. $\int \frac{3x^2}{\sqrt{x^3+1}}\,dx =$ (A) $2\sqrt{x^3+1}+C$ (B) $\frac{3}{2}\sqrt{x^3+1}+C$ (C) $\sqrt{x^3+1}+C$ (D) $\ln\sqrt{x^3+1}+C$ (E) $\ln(x^3+1)+C$

20. $\int \tan^3 x \sec^2 x\,dx =$ (A) $\frac{\tan^2 x}{2}+\frac{\sec^4 x}{4}+C$ (B) $\frac{\tan^4 x}{4}+\frac{\tan^2 x}{2}+C$

(C) $\frac{\tan^4 x}{4}+C$ (D) $\frac{\sec^2 x}{2}-\frac{\sec^4 x}{4}+C$ (E) $\frac{\tan^4 x}{4}+\frac{\sec^3 x}{3}+C$

PART II. (40 points) Answer all questions.

1. Given the function $f(x)=\frac{6(x^2-16)}{x^2-9}$ [14 points]

(a) Use the first derivative to determine in which intervals the function is increasing.
(b) Use the second derivative to determine in which intervals the curve is concave up.
(c) Sketch and LABEL the graph of y = f(x) in the interval $-10 \le x \le 10$. LABEL all local extrema, points of inflection, and asymptote lines.

2. A particle is moving along the x-axis with velocity $v(t) = 3t^2 - 10t - 8$ ft/sec where $t \ge 0$. [13 points]
(a) Find the position as a function of time if x(2) = 4.
(b) Find the total distance traveled from t = 1 second until t = 5 seconds.

3. A rectangle is bounded between the x-axis and the ellipse $y=\sqrt{16-4x^2}$
(a) What length and width should the rectangle have so that the area is a maximum?
(b) When a particle is at $(1, 2\sqrt{3})$, it is moving downward along the ellipse (dy/dt) at the rate of 2 cm per second. At that instant, find the rate of change of the position in a horizontal direction (dx/dt).

[13 points]