ROCKET POWER ANOMALY

May 6, 2007 By Tom Schum Note: The anomaly is explained on pages 4-9.

For fun, we will analyze the operation of a rocket. Remember this is entirely math; no experiments were done. If the real world does not match this math, we will probably not be able to tell.

The rocket, with fuel, will have a mass of 1 kg. The payload will include the rocket and the rocket motor, and will have a mass of 0.1 kg. The rest of it (0.9 kg) will be fuel. The rocket motor will be a linear accelerator that applies 1 newton of force to a 1 milligram pellet for 0.001 seconds, accelerating and ejecting 1000 pellets every second. This works out to 1 gram of ejected pellets every second (the ejecta). Powered flight lasts 900 seconds, until the rocket motor ejects the last pellet.

We will employ interval math to do this analysis. The interval will be 0.001 second. During our analysis we will take into account what happens with the rocket, and with the ejecta.

Initially the rocket is at rest in free space. Then we accelerate the first pellet: An applied force of 1 newton will accelerate a milligram to 1 million m/sec in one second. But we are accelerating the pellet for 0.001 second, so it emerges from the accelerator at a speed of 1000 m/sec after moving only 0.5 meters. As the pellet enters the accelerator and begins to be accelerated, the effective mass of the rocket is reduced by the mass of the pellet, and the acceleration force of 1 newton is felt by the lightened load the rocket has become. This is because when the pellet enters the accelerator it becomes the "reaction mass".

Just for fun, let's make a little mistake when we load the fuel into the rocket so we end up loading an extra pellet. Now, the mass of the rocket becomes 1.000001 kg. When we fire the first pellet, we will apply 1 newton of force to 1 kg of mass (the rocket) for 0.001 seconds. Our second pellet will be accelerated in the next time period of 0.001 seconds, and we will apply 1 newton of force to 0.999999 kg of mass for 0.001 second. And so we go, until the fuel is all gone. The last pellet will be fired from a rocket depleted of fuel, having only 0.1kg of mass, making interval 900,001 the final interval. Notice how much less mass the rocket has at the end of powered flight, compared to the beginning, but thrust force is unchanged.

We want to keep track of (**J**) how far the rocket has traveled up to now, (**K**) how fast the rocket is moving as the pellet enters the accelerator, (**L**) the mass of the rocket after the pellet enters the accelerator, (**N**) the speed of the rocket after the pellet leaves the accelerator, (**O**) the speed of the pellet as it leaves the accelerator and (**P**) the distance traveled by the rocket during this interval.

Let's look at this set of data one entry at a time:
J. How far has the rocket traveled up to now? All we know is that at the beginning it was not moving.
K. How fast is the rocket moving as the pellet enters the accelerator? At the beginning, speed is zero.
L. What is the mass of the rocket after the pellet enters the accelerator? At the beginning, it is 0.000001 kg less than the total mass of the rocket.
N. What is the speed of the rocket as the pellet leaves the accelerator? The accelerator has accelerated the rocket, so the rocket is not at rest, even during the first interval. We have to calculate the acceleration for this interval, and add it to the speed of the rocket at the beginning of this interval to find a resultant speed: [(force/mass) x 0.001 second] + **K** (speed of rocket at beginning of this interval)
O. (the letter "**O**") What is the speed of the pellet as it leaves the accelerator? To find this we have to take into account how fast the rocket is moving, and then the pellet is moving 1000 m/s in the opposite direction. For the speed of the rocket, we will use item **K** above ("How fast is the rocket moving as the pellet enters the accelerator?"). Then we will subtract 1000 m/s.
P. How far has the rocket moved during this interval? We will take the average speed of the rocket during this interval and multiply it by the duration of the interval (0.001 sec) to get the distance increment for this interval. Average speed is the sum of item **K** plus item **N**, divided by two.

Now we need to take the results for this interval and fill in the starting values for the next interval:
J. How far has the rocket traveled up to now? Add items **J** and **P**.

K. How fast is the rocket moving as the pellet enters the accelerator? This is item **N**.
L. What is the mass of the rocket after the pellet enters the accelerator? Subtract 0.000001 kg from the number we just used in item **L**.

After every 5000 intervals (an arbitrary subdivision of the total 900,001 intervals), we will want to look at the data and make some graphs. We need to remember all the data we calculated, and record some averages for each of these five-second time periods. There will be 180 sets of this data:
G. Increment number (just so we can keep track of the time)
B. Average mass of rocket : item **L** added together for these 5000 intervals, then divide by 5000.
C. Average speed of rocket : item **N** added together, then divide by 5000.
D. Distance moved by rocket during subdivision : add item **P** together, but do not divide by 5000.
E. Average speed of ejecta : item **O** added together, then divide by 5000. Mass will always be 5 grams.
These averages will be recalculated from zero for every 5000 intervals, so that each dataset represents 5 seconds of flight.

It is possible to use the figures from the intervals (interval 5000, interval 10,000, interval 15,000 etc.) as the graph data. However, averaging the data as a whole includes **all** the data.

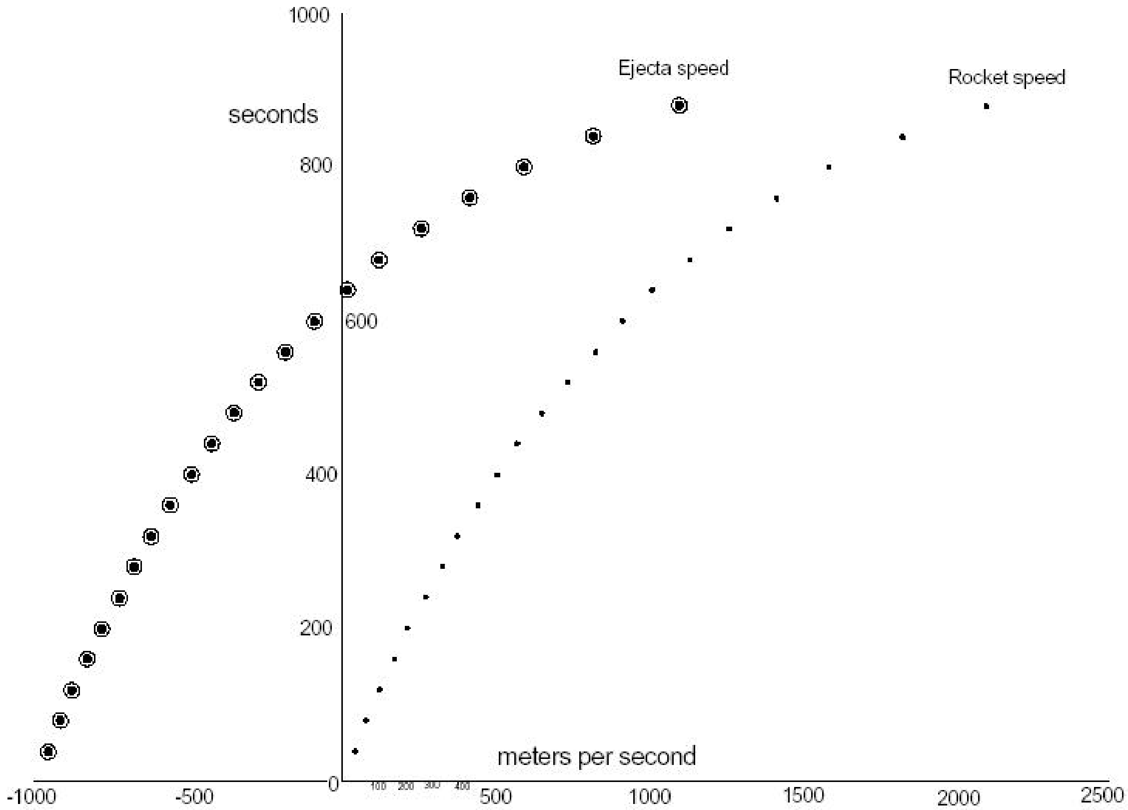
See Appendix A for the HP-33s calculator program used to produce the following averaged data.

Interval number **RCL G** milliseconds	Rocket mass **RCL B** kilograms	Rocket speed **RCL C** meters/second	Distance moved **RCL D** meters	Pellet speed **RCL E** meters/second
5,000 (5 sec)	0.9975005	2.504677	12.52088	-997.4963
10,000 10s	0.9925005	7.529824	37.64660	-992.4712
15,000 15s	0.9875005	12.58035	62.89922	-987.4207
20,000 20s	0.9825005	17.65651	88.28003	-982.3445
25,000 25s	0.9775005	22.75858	113.7903	-977.2424
30,000 30s	0.9725005	27.88680	139.4315	-972.1142
35,000 35s	0.9675005	33.04147	165.2048	-966.9596
40,000 40s	0.9625005	38.22284	191.1116	-961.7782
45,000 45s	0.9575005	43.43119	217.1534	-956.5699
50,000 50s	0.9525005	48.66682	243.3315	-951.3342
55,000 55s	0.9475005	53.93000	269.6474	-946.0711
60,000 60s	0.9425005	59.22103	296.1025	-940.7800
65,000 65s	0.9375005	64.54021	322.6984	-935.4609
70,000 70s	0.9325005	69.88783	349.4364	-930.1132
75,000 75s	0.9275005	75.26420	376.3183	-924.7369
80,000 80s	0.9225005	80.66963	403.3454	-919.3315
85,000 85s	0.9175005	86.10444	430.5195	-913.8967
90,000 90s	0.9125005	91.56894	457.8420	-908.4322

Looking at just the first 90 seconds of operation, the averages seem to make sense. Average mass, accelerated with a force of 1 newton, produces in five seconds a speed increment as shown. The average speed for 5 seconds causes the rocket to move thru the distance as shown.
Pellet speed shows the effect of the average speed of the rocket for that 5-second subdivision. If you take average pellet speed and add it to 1000 m/sec, the answer will be closely equal to the average rocket speed for that subdivision. See Appendix C for the rest of the data.

Notice nothing exactly matches, but it's close. Keep in mind that these averages are not used in the main calculation loop. The main loop proceeds in 1 millisecond increments independently.

Let's try a graph of rocket speed and pellet speed (ejecta speed) vs time:



It is fairly easy to see that at a rocket speed of 1000 m/sec, ejecta speed is zero. This occurs at about 640 seconds into the flight. Ejecta speed is always about 1000 m/sec less than the rocket speed. What does this tell us? Very little!

What is to be made of all this information?
What is the "Rocket Power Anomaly"?

Well, as the rocket mass goes down the effectiveness of the thrust increases. This is because the thrust has less mass to work against. Is there anything exceptional about this? No!

Power: The Rocket Power Anomaly Explained

How much power does the rocket motor in our little simulation actually deliver? As the mass of the rocket reacts to the force exerted on the propellant pellet by the rocket motor, the rocket moves. This means the specification of the rocket motor we are using is not a "constant power" specification. Our rocket motor applies 1 newton to 1 milligram for 1/1000 second, and it does this 1000 times per second. The rocket, with its mass reduced 1000 times per second, reacts more and more to the motor force. **This rocket motor must increase its power as the mass of the rocket decreases**, because it moves the opposite direction in reaction to the force it is applying to accelerate the pellet. This is because the accelerator (the rocket motor) **is part of the rocket**.

The HP-42s emulator was used to extract the following data. See Appendix B for the program.

Starting near the beginning, with interval 5000:

08	5000 (we are looking at results for interval 5000)
10	12.515867 meters (distance moved by rocket up to the beginning of this interval)
11	5.0115343 m/sec (rocket speed at beginning of this interval)
12	0.995002 kg (rocket mass at beginning of this interval)
13	0.995001 kg (rocket mass to be accelerated)
14	0.0010050241 m/sec (change in rocket speed in this interval)
15	5.0125393 m/sec (rocket speed at end of this interval)
16	-994.98847 m/sec (speed of ejected pellet)
17	0.0050120368 meters (distance rocket moves during this interval)

08	500,000 (we are looking at results for interval 500,000)
10	153,425.62 meters (distance moved by rocket up to the beginning of this interval)
11	693.14468 m/sec (rocket speed at beginning of this interval)
12	0.500002 kg (rocket mass at beginning of this interval)
13	0.500001 kg (rocket mass to be accelerated)
14	0.001999996 m/sec (change in rocket speed in this interval)
15	693.14668 m/sec (rocket speed at end of this interval)
16	-306.85532 m/sec (speed of ejected pellet)
17	0.69314568 meters (distance rocket moves during this interval)

08	900,000 (we are looking at results for interval 900,000)
10	669,738.49 meters (distance moved by rocket up to the beginning of this interval)
11	2,302.5706 m/sec (rocket speed at beginning of this interval)
12	0.10000200 kg (rocket mass at beginning of this interval)
13	0.10000100 kg (rocket mass to be accelerated)
14	0.0099999000 m/sec (change in rocket speed in this interval)
15	2,302.5806 m/sec (rocket speed at end of this interval)
16	1,302.5706 m/sec (speed of ejected pellet)
17	2.3025756 meters (distance rocket moves during this interval)

To find total distance for the W = FD formula (Work = Force x Distance), we must add the distance over which the pellet was accelerated (0.5 meters) to the distance moved by the rocket in reaction to the acceleration force. We then know how far the pellet was actually pushed (relative to the rocket) in order to use 1 newton of acceleration force for 0.001 second.

In interval 5000, this distance is 0.5050120368 meters. Total energy is 0.5050120368 joules, or 505.0120368 watts (if 1000 intervals are all the same as this one). In other words, to accelerate the pellet takes 505 watts, not just 500 watts. The excess 5 watts goes to increase the rocket momentum. We calculate the distance moved by the rocket as the average speed of the rocket during the interval, multiplied by the time duration of the interval. This distance is incorrect, as we will see on page 7.

In interval 393,470 (see page 7), the distance is 0.99999994 meters, and total energy is 0.99999994 joules. Instantaneous power at this moment is 999.99994 watts. 500 watts goes into the pellet, and 499.99994 watts goes into the rocket momentum.

In interval 500,000, the distance is 1.19314568 meters, and total energy is 1.19314568 joules. Instantaneous power at this moment is 1193.14568 watts. 500 watts goes into the pellet, and 693 watts goes into the rocket momentum.

In interval 900,000, the distance is 1.8025756 meters, and total energy is 1.8025756 joules. Instantaneous power at this moment is 1802.5756 watts. 500 watts goes into the pellet, and 1302.6 watts increases the momentum of the rocket.

The calculator program states that the pellet will leave the accelerator at a speed of -1000 m/sec relative to the speed the system (rocket with pellet) was moving at the beginning of the interval. The accelerator moves with the rocket, so it must accelerate the pellet further than it does when the rocket is at rest. As the speed of the rocket increases, the real power applied increases accordingly.

By the "rules of Kinetic Energy calculation" the distance displaced by the rocket during the interval must be added to the pellet acceleration distance, to get the entire distance. This entire distance determines the energy needed during the interval.

So, we have specified our rocket motor incorrectly. We should have said that the power delivered or used by our rocket motor is always 500 watts.

This would have changed the interval math quite a bit. The pellet speed would always be -1000 m/sec relative to the average speed of the rocket during the interval (not the speed of the rocket at the beginning of the interval).

Calculation of this kind of interval would require an iterative algorithm and might proceed much more slowly than the existing program. However, one might expect results to match reality more closely in this case. We see now that a "rocket power anomaly" exists as an artifact of the mathematical simulation we are using.

A corrected simulation will not deliver a constant force to the rocket during the flight, since the power delivered by the rocket motor must be constant. As the flight progresses, the delivered acceleration force must decrease for the power to remain constant.

The power seen in the original simulation is real (real in the mathematical sense only), and is required if the acceleration force to the rocket is held constant. We see that this requires the power delivered by the rocket motor to increase dramatically. This increasing power as the rocket burn time progresses is a design factor in solid-fuel boosters, since the burn increases the area of propellant involved in the thrust reaction inside the rocket motor. Liquid-fueled boosters are not able to do this.

KINETIC ENERGY AND MOMENTUM

Careful analysis of the data shows that momentum and kinetic energy may be simultaneously conserved during our rocket flight. That is, it is possible to find a **plausible explanation** for the changes in kinetic energy as the rocket flight proceeds.

Using the data from the HP-42s emulator (see appendix B) we have:
08 995,000 (we are looking at results for interval 995,000)
10 968,500.96 meters (distance moved by rocket up to the beginning of this interval)
11 5298.0179 m/sec (rocket speed at beginning of this interval)
12 0.005002 kg (rocket mass at beginning of this interval)
13 0.005001 kg (rocket mass to be accelerated)
14 0.19996001 m/sec (change in rocket speed in this interval)
15 5298.2179 m/sec (rocket speed at end of this interval)
16 4298.0179 m/sec (speed of ejected pellet)
17 5.2981179 meters (distance rocket moves during this interval)

KE of rocket at beginning of interval, after pellet enters the accelerator = 70,186.519 J
KE of pellet at beginning of interval, as it enters the accelerator = 14.034497 J
Total system KE before acceleration = 70,200.553 Joules.
After 0.001 sec, we reach the end of the interval.
KE of rocket at end of interval, after it has been accelerated = 70,191.817 J
KE of pellet at end of interval, after it has been accelerated = 9.236479 J total = 70.201053 joules
Total system KE after acceleration = 70,201.053 joules.
Difference in total KE = 0.499982 J
Energy required to accelerate pellet during interval = 0.5000301 J
So, the books seem to balance here.
We use some energy to change the KE of the system. All these changes add to the system's KE.

Will you find this balance working in any interval you choose? Probably so.

Let's try interval 200,000
08 200,000 (we are looking at results for interval 200,000)
10 21,484.924 meters (distance moved by rocket up to the beginning of this interval)
11 223.14218 m/sec (rocket speed at beginning of this interval)
12 0.800002 kg (rocket mass at beginning of this interval)
13 0.800001 kg (rocket mass to be accelerated)
14 0.0012499984 m/sec (change in rocket speed in this interval)
15 223.14343 m/sec (rocket speed at end of this interval)
16 -776.85782 m/sec (speed of ejected pellet)
17 0.2231428 meters (distance rocket moves during this interval)

KE of rocket at beginning of interval, after pellet enters the accelerator = 19,916.997 J
KE of pellet at beginning of interval, as it enters the accelerator = 0.024896215 J
Total system KE before acceleration = 19,917.022 Joules.
After 0.001 sec, we reach the end of the interval.
KE of rocket at end of interval, after it has been accelerated = 19,917.22 J
KE of pellet at end of interval, after it has been accelerated = 0.30175404 J
Total system KE after acceleration = 19,917.522 joules.
Difference in total KE = 0.5 J
Energy required to accelerate pellet during interval = 0.5 J (this is always the same)
So, the books seem to balance here.
We use some energy to change the KE of the system. All these changes add to the system's KE.

Let's try interval 393,470
08 393,470 (we are looking at results for interval 393,470)
10 90,203.787 meters (distance moved by rocket up to the beginning of this interval)
11 499.99911 m/sec (rocket speed at beginning of this interval)
12 0.606532 kg (rocket mass at beginning of this interval)
13 0.606531 kg (rocket mass to be accelerated)
14 0.0016487203 m/sec (change in rocket speed in this interval)
15 500.00076 m/sec (rocket speed at end of this interval)
16 -500.00089 m/sec (speed of ejected pellet)
17 0.49999994 meters (distance rocket moves during this interval)

KE of rocket at beginning of interval, after pellet enters the accelerator = 75,816.106 J
KE of pellet at beginning of interval, as it enters the accelerator = 0.12499956 J
Total system KE before acceleration = 75,816.231 Joules.
After 0.001 sec, we reach the end of the interval.
KE of rocket at end of interval, after it has been accelerated = 75,816.606 J
KE of pellet at end of interval, after it has been accelerated = 0.12500044 J
Total system KE after acceleration = 75,816.731 joules.
Difference in total KE = 0.50000082 J
Energy required to accelerate pellet during interval = 0.5 J (this is always the same)

How much work has been done to accelerate the pellet? In this simulation the pellet will be accelerated to a speed of -1000 m/sec relative to the speed of the rocket + pellet at the beginning of the interval. We now calculate the work done in the inertial frame that is moving at the speed of the rocket + pellet at the beginning of the interval. The work done is FD, force multiplied by distance. The pellet mass to be accelerated is 0.000001 kg and the acceleration force is -1 newton, so the acceleration is F/M (Force divided by Mass) = -1,000,000 m/sec/sec. Using the distance formula D=1/2(a)(t^2), we get a distance of -0.5 meters. To accelerate the 1 mg pellet to a speed of -1000 m/sec using a force of 1 newton takes 0.001 second, and the pellet moves a distance of -0.5 meters in this acceleration process. Total work done on the pellet is 1 newton multiplied by 0.5 meters = 0.5 joule.

How much work is done to accelerate the rocket? The rocket mass is subjected to 1 newton of acceleration force for 0.001 seconds during the interval. In interval 393,470, the rocket mass to be accelerated is 0.606531 kg, so the acceleration is 1.6487203 m/sec/sec. We are calculating the work done in the inertial frame that is moving at the speed of the rocket + pellet at the beginning of the interval. The work done is FD, force multiplied by distance. Using the distance formula D=1/2(a)(t^2), we get a distance of 0.00000082436017 meters (8.2436017e-7). This distance is 1/606,531 of the distance moved by the pellet. The rocket mass is greater than the pellet mass by a factor of 606531. The distance ratio is the inverse. Regardless, the rocket moves 8.2436017e-7 meters, and the work done is 8.2436017e-7 joules.

Total work done in the interval is the sum of the work done on the pellet and the work done on the rocket. This total is 0.50000082 joules. Notice that this number is EXACTLY the same as the difference in total KE above, and is a small amount MORE than the energy required to accelerate the pellet. This number exactly balances the KE difference for this interval. So, in every new interval we can work in a new frame of reference to correctly calculate the work done by the rocket motor.

It is equally valid to calculate the change in KE of the rocket using the distance moved in the rest frame:
KE of rocket at beginning of interval, after pellet enters the accelerator = 75,816.1064893 J
KE of rocket at end of interval, after it has been accelerated = 75,816.6064892 J
Difference in KE of rocket = 0.499999938961 joules (requires min 17 digits of precision in our calculator).
Total distance moved by rocket during interval (register 17) = 0.499999938961 (this is an exact match).
The difference between the energy added to the rocket and the total energy used is 8.85398891705e-7 joules. The difference in the KE of the pellet is 8.85398891705e-7 joules (this is an exact match).

Let's try this in interval 5000:

08	5000 (we are looking at results for interval 5000)
10	12.5158672661 meters (distance moved by rocket up to the beginning of this interval)
11	5.01153428687 m/sec (rocket speed at beginning of this interval)
12	0.995002 kg (rocket mass at beginning of this interval)
13	0.995001 kg (rocket mass to be accelerated)
14	1.00502411555e-3 m/sec (change in rocket speed in this interval)
15	5.01253931098 m/sec (rocket speed at end of this interval)
16	-994.988465713 m/sec (speed of ejected pellet)
17	5.01203679892e-3 meters (distance rocket moves during this interval)

Kinetic energy of rocket at beginning of this interval after the pellet has entered the accelerator but before the accelerator has started = rocket mass to be accelerated (register 13) multiplied by the square of the speed of the rocket at the beginning of the interval (register 11), divided by two = 12.4949618222 joules.

Kinetic energy of rocket at end of interval, after acceleration = same rocket mass (register 13) multiplied by the square of the speed of the rocket at the end of the interval (register 15), divided by two = 12.499973859 joules.

The difference in KE of the rocket is 5.01203679892e-3 joules.

Using the total distance moved by the rocket during this interval (register 17) we calculate the kinetic energy added to the rocket as 5.01203679892e-3 joules.
It's an exact match!

KE of pellet at beginning = 1.25577379542e-5 joules
KE of pellet at end = 4.95001023451e-1 joules
Difference in pellet KE = 4.94988465713e-1 joules has been added to the initial value of pellet KE.

If we add the difference in rocket KE to the difference in pellet KE, we get 5.00000502512e-1 joules.

Now, we work in the frame that is moving at the initial velocity of the rocket:
Acceleration = 1 newton/rocket mass = 1.00502411555 m/sec/sec
D=1/2(a)(t^2) = 5.02512057777e-7 meters. This is how far the rocket moves as it accelerates.
Add 0.5 meters (the distance moved by the pellet as it accelerates) and we get 5.00000502512e-1.
Work = FD (force multiplied by distance) = 1 newton x 5.00000502512e-1 = 5.00000502512e-1 joules.

Again, an exact match shows that calculating the energy in the moving frame is equivalent to calculating the energy in the rest frame.

Let's try this in interval 995,000:

08	995,000 (we are looking at results for interval 995,000)
10	968,500.963424 meters (distance moved by rocket up to the beginning of this interval)
11	5,298.01790987 m/sec (rocket speed at beginning of this interval)
12	0.005002 kg (rocket mass at beginning of this interval)
13	0.005001 kg (rocket mass to be accelerated)
14	1.99960007998e-1 m/sec (change in rocket speed in this interval)
15	5,298.21786988 m/sec (rocket speed at end of this interval)
16	4,298.01790987 m/sec (speed of ejected pellet)
17	5.29811788988 meters (distance rocket moves during this interval)

KE of rocket at beginning = 70,186.5189302 joules (register 13=mass, register 11=speed)
KE of rocket at end = 70,191.8170481 joules (register 13=mass, register 15=speed)
Difference in rocket KE = 5.29811788988 joules

Register 17 multiplied by acceleration force = 5.29811788988 joules.
Again, it's an exact match.

Pellet KE at beginning = 14.0344968867 joules
Pellet KE at end = 9.2364789768
Difference in pellet KE = -4.79801790987.
4.79801790987 joules has been subtracted from the KE of the pellet.

Now we add the differences to get 5.00099980004e-1 joules. This is the total work done during this interval.

Calculating in the frame that moves at the initial speed:
Rocket acceleration = 1 newton divided by rocket mass = 199.960007998 m/sec/sec
Rocket distance D=1/2(a)(t^2) = 9.99800039992e-5 meters
Adding pellet distance of 0.5 meters to this = 5.00099980004e-1 meters
Work = FD = 1 newton x total distance = 5.00099980004e-1 joules

Again, an exact match illustrates that calculating KE can be done in either the rest frame or the frame that moves at the initial speed of the rocket at the beginning of the interval.

Toward a Clearer View of Kinetic Energy

Viewing the impulse as the only energy input to the system, we can view the change in kinetic energy of the mass as related only to the impulse. This is very different from the currently preferred view, but we have seen above that the currently preferred view also works.

Currently, the change in kinetic energy takes into account the pre-existing state of motion of the mass, before the impulse was applied, and makes it a significant part of the change in total kinetic energy. The pre-existing state of motion of the mass, in this case, has nothing to do with the impulse we apply. The pre-existing state of motion of the mass will remain unchanged if we do not apply an impulse.

It is potentially misleading to use the pre-existing state of motion in the calculation of final kinetic energy, even though this method is equivalent.

It is not at all misleading to calculate the change in kinetic energy based on only the impulse applied in the moving frame in which the impulse acts.

APPENDIX A:

Calculator program for the HP 33s RPN programmable calculator

(Typical HP-33s calculator cost is $45 as of April, 2007 on amazon.com)

Register assignments:

A	Subdivision interval number
B	Average rocket mass during subdivision
C	Average rocket speed during subdivision
D	Distance moved by rocket during subdivision (not total distance)
E	Average speed of ejecta during subdivision
F	Pellet mass (preset to 0.000001 kg)
G	Interval number (global)
H	Force generated by rocket motor (preset to 1 newton)
I	Final interval number (preset to 900,001)
J	Distance moved by rocket as this interval begins
K	Speed of the rocket as this interval begins
L	Mass of the rocket after pellet enters accelerator (preset to 1.000001 kg at start of program)
M	Subdivision size in number of intervals (preset to 5000)
N	Speed of rocket after pellet has been accelerated
O	Speed of pellet as it is ejected from accelerator
P	Distance moved by rocket during this interval
V	Time duration of interval (preset to 0.001 sec)

Program:

```
LBL T        Initialize (hit this before you start the simulation)
0            numeric value =0
STO A        Store 0 in register A (subdivision increment number)
STO B        Store 0 in subdivision rocket mass accumulator
STO C        Store 0 in subdivision rocket speed accumulator
STO D        Store 0 in subdivision distance accumulator
STO E        Store 0 in subdivision pellet speed accumulator
STO G        Store 0 as increment number (global)
STO K        Store 0 as rocket speed at beginning
STO N        Store 0 as new rocket speed at end of interval
STO O        Store 0 as pellet speed as it is ejected
STO P        Store 0 as distance moved by rocket during this interval
1.000001     mass of rocket at beginning, in kg
STO L        Store in register L
RTN          end of subroutine T, return to user control
```

```
LBL X       The main calculation loop (hit this to start simulation)
1           numeric value =1
STO+ G      Add 1 to the increment number
STO+ A      Add 1 to the subdivision increment number
RCL F       Recall pellet mass ( 0.000001 kg )
STO- L      Subtract pellet mass from rocket mass
RCL H       Recall acceleration force ( 1 newton )
RCL L       Recall mass of rocket to be accelerated
STO+ B      Add this mass to subdivision mass accumulator
DIVIDE      Divide acceleration by mass
RCLx V      Multiply by stored interval time period ( 0.001 sec )
                  This is the speed increment for the interval
RCL+ K      Add to speed of rocket at beginning of interval
STO N       Store value as final speed of rocket at end of this interval
STO+ C      Add this final speed to subdivision speed accumulator
RCL K       Recall speed of rocket at beginning of interval
RCL- W      Subtract stored value for pellet velocity ( 1000 m/sec )
STO O       Store as pellet speed for this interval
STO+ E      Add this speed to subdivision pellet speed accumulator
RCL N       Recall speed of rocket at end of this interval
RCL+ K      Add stored value of rocket speed at beginning of interval
2           numeric value =2
DIVIDE      Divide speed sum by 2 to get average rocket speed
RCLx V      Multiply by stored interval time period ( 0.001 sec )
                  The rocket moves this distance during this interval
STO P       Store incremental distance for this interval
STO+ D      Add this distance to subdivision distance accumulator
RCL I       Recall maximum interval number ( 900,001 )
RCL G       Recall current interval number
x>=y        If current interval number is same as max, do next step,
                  Otherwise, skip it and jump to the following step
RTN         Stop: we are done with the entire simulation
RCL M       Recall subdivision increment number ( 5000 )
RCL A       Recall current subdivision increment number
x>=y        If current is same as stored value, do next step,
                  Otherwise, skip it and jump to the following step
GTO Q       Process accumulators for user

LBL S       Prepare for next interval
RCL P       Recall incremental distance
STO+ J      Add it to stored total distance moved by rocket
RCL N       Recall speed of rocket at end of interval
STO K       Store it as speed of rocket at beginning of next interval
GTO X       Go to the beginning of LBL X
RTN         End of subroutine S

LBL Z       Run next subdivision (hit this for every new dataset)
0           numeric value =0
STO A       Store 0 in subdivision increment number
STO B       Store 0 in subdivision rocket mass accumulator
STO C       Store 0 in subdivision rocket speed accumulator
STO D       Store 0 in subdivision distance accumulator
STO E       Store 0 in subdivision pellet speed accumulator
GTO S       Go to prepare for next interval
RTN         End of subroutine Z
```

```
LBL Q       Prepare subdivision accumulators for user
RCL B       Recall rocket mass accumulator
RCL M       Recall subdivision size ( 5000 intervals )
DIVIDE      Calculate average rocket mass in subdivision
STO B       Store it as average rocket mass
RCL C       Recall rocket speed accumulator
RCL M       Recall subdivision size ( 5000 intervals )
DIVIDE      Calculate average rocket speed in subdivision
STO C       Store it as average rocket speed
RCL E       Recall subdivision pellet speed accumulator
RCL M       Recall subdivision size ( 5000 intervals )
DIVIDE      Calculate average pellet speed in subdivision
STO E       Store it as average pellet speed
RTN         End of subroutine Q, user may read registers now
```

<u>A few miscellaneous notes about the program</u>:

1. You need to initialize numerous registers at the beginning. Subroutine **T** merely provides an easy way to re-start if you think you have made some mistake.
2. After **XEQ T** (executes subroutine **T**), you must start the main loop with **XEQ X.**
3. The main loop takes about 10 minutes to churn thru 5000 intervals.
4. When execution stops, you must manually read and record the values of registers **G**, **B**, **C**, **D**, **E**. This is most easily done using the **RCL** command (**RCL G**, **RCL B** etc.)
5. To start the next subdivision, hit **XEQ Z**. Then wait about 10 minutes for the next dataset.
6. Keep an extra set of fresh batteries available (2x CR2032 lithium coin cells). Memory is **not** lost when you change batteries, if you follow replacement instructions with care.

APPENDIX B:

PROGRAM FOR WINDOWS Free42 Decimal CALCULATOR EMULATOR

Download this freeware from `http://home.planet.nl/~demun000/thomas_projects/free42/`
If you like this emulator, send Thomas Okken $30 via Paypal.
This emulator runs a million intervals in 2 minutes on a 2 GHz pentium, to at least 20 digits of precision.
The user interface graphic is a high-res view of the actual calculator keyboard, and is excellent.
My HP-42s calculator will also run the following program, but 1000x more slowly.

To run this program:

1. Enter all the program steps listed below into the calculator emulator, or import the program file.
    Note program files are generated by the calculator emulator only, and are not text files.
    To enter program steps, first hit the orange key then the "PRGM" key.
    To access the `LBL, RTN`, and other functions, hit the orange key then the "PGM.FCN" key.
    To exit programming mode, first hit the orange key then the "PRGM" key.
2. Set display parameters: hit the orange key then "DISP", then select scientific notation etc.
3. **`XEQ "INIT"`** to clear and initialize the necessary registers, and set default stop value = 5000.
4. Key in the interval number you are interested in.
    Range is between 1 and 1,000,000. Interval 1,000,001 will produce an error.
5. **`XEQ "LAST"`** to set your preferred stop value.
    Note other values such as start mass may be changed by directly accessing the registers.
6. **`XEQ "X"`** to run the calculation. The calculator screen will show you that the program is running.
7. When the calculation finishes you may inspect the registers.
    For example, **`RCL 10`** shows you total distance moved up to the beginning of the stop interval.
8. To see results for the next interval after the stop, use **`XEQ "NEXT"`**.
    This will run another single interval then it will stop, because of the test in program step 51.

Register Assignments:

08	Current interval
09	Last interval (this is where we stop)
10	Distance total at beginning of this interval
11	Speed of rocket at beginning of this interval
12	Mass of rocket at beginning of this interval
13	Mass of rocket while it is accelerating during this interval
14	Speed increment (change in rocket speed during this interval)
15	Speed of rocket at end of this interval
16	Speed of ejected pellet
17	Distance increment (distance traveled during this interval)

Constants built into the program:

Acceleration force = 1 newton
Interval duration = 0.001 second
Pellet mass = 1 mg = 0.000001 kg
Rocket mass at start of calculation = 1.000001 kg

Program listing:

```
00 { 125-Byte Prgm }
01>LBL "INIT"
02 0              numeric value = 0
03 STO 08         store 0 in all the registers we intend to use
04 STO 09
05 STO 10
06 STO 11
07 STO 12
```

```
08 STO 13
09 STO 14
10 STO 15
11 STO 16
12 STO 17
13 1.000001       numeric value = 1.000001
14 STO 12        Rocket mass at start of calculation
15 5E3           default last interval is 5000
16 STO 09        store last interval
17 RTN           end of subroutine "INIT"
18>LBL "LAST"    Allow easy storage of a new "last interval" value
19 STO 09        store last interval
20 RTN           end of subroutine "LAST"
21>LBL "X"       MAIN CALCULATION LOOP
22 RCL 08        recall current interval
23 1             numeric value = 1
24 +             add 1 to current interval number
25 STO 08        store new value as current interval
26 RCL 12        recall rocket mass at beginning of this interval
27 1E-6          numeric value 0.000001 = mass of pellet
28 -             subtract pellet mass from rocket mass
29 STO 13        store mass of rocket being accelerated
30 1/X           1 newton divided by mass = acceleration
31 1E-3          numeric value = 0.001 = duration of interval
32 ×             acceleration multiplied by time = speed change
33 STO 14        store rocket speed increment
34 RCL 11        recall rocket speed at beginning of interval
35 +             add speed increment and beginning speed
36 STO 15        store rocket speed at end of this interval
37 RCL 11        recall rocket speed at beginning of interval
38 1E3           numeric value = 1000 = relative ejecta speed
39 -             subtract ejecta speed from beginning speed
40 STO 16        store ejecta speed for this interval
41 RCL 11        recall rocket speed at beginning of interval
42 RCL 15        recall rocket speed at end of interval
43 +             add the two speeds together
44 2             numeric value = 2
45 ÷             divide by 2 to get "arithmetic average" speed
46 1E-3          numeric value = 0.001 = duration of interval
47 ×             multiply by average speed to get distance increment
48 STO 17        store distance moved during this interval
49 RCL 09        recall stopping point (last interval number)
50 RCL 08        recall current interval number
51 X>=Y?         is current interval equal to or greater than stop value?
52 RTN           calculation is finished, return to user control
53>LBL "NEXT"    if current interval is less than stop value, continue
54 RCL 15        recall rocket speed at end of interval
55 STO 11        store as rocket speed at beginning of interval
56 RCL 17        recall rocket distance moved
57 RCL 10        recall total distance so far
58 +             add these two distances to get a new total distance
59 STO 10        store new total distance so far
60 RCL 13        recall rocket mass while accelerating during this interval
61 STO 12        store as new value of rocket mass before acceleration
62 XEQ "X"       continue to calculate values for new interval
63 RTN           end of subroutine "NEXT"
64 .END.         end of program listing.
```

Sample calculation results from the HP-42s emulator:

08	1,000,000 (we are looking at results for interval 1,000,000)
10	999,978.91 meters (distance moved by rocket up to the beginning of this interval)
11	13,392.727 m/sec (rocket speed at beginning of this interval)
12	2 milligrams (rocket mass at beginning of this interval)
13	1 milligram (rocket mass to be accelerated)
14	1000 m/sec (change in rocket speed in this interval)
15	14,392.727 m/sec (rocket speed at end of this interval)
16	12,392.727 m/sec (speed of ejected pellet)
17	13.892727 meters (distance rocket moves during this interval)

08	900,000 (we are looking at results for interval 900,000)
10	669,738.49 meters (distance moved by rocket up to the beginning of this interval)
11	2,302.5706 m/sec (rocket speed at beginning of this interval)
12	0.10000200 kg (rocket mass at beginning of this interval)
13	0.10000100 kg (rocket mass to be accelerated)
14	0.0099999000 m/sec (change in rocket speed in this interval)
15	2,302.5806 m/sec (rocket speed at end of this interval)
16	1,302.5706 m/sec (speed of ejected pellet)
17	2.3025756 meters (distance rocket moves during this interval)

08	500,001 (we are looking at results for interval 500,001)
10	153,426.31 meters (distance moved by rocket up to the beginning of this interval)
11	693.14668 m/sec (rocket speed at beginning of this interval)
12	0.50000100 kg (rocket mass at beginning of this interval)
13	0.50000000 kg (rocket mass to be accelerated)
14	0.0020000000 m/sec (change in rocket speed in this interval)
15	693.14868 m/sec (rocket speed at end of this interval)
16	-306.85332 m/sec (speed of ejected pellet)
17	0.69314768 meters (distance rocket moves during this interval)

08	500,002 (we are looking at results for interval 500,002)
10	153,427.01 meters (distance moved by rocket up to the beginning of this interval)
11	693.14868 m/sec (rocket speed at beginning of this interval)
12	0.50000000 kg (rocket mass at beginning of this interval)
13	0.49999900 (rocket mass to be accelerated)
14	0.0020000040 m/sec (change in rocket speed in this interval)
15	693.15068 m/sec (rocket speed at end of this interval)
16	-306.85132 m/sec (speed of ejected pellet)
17	0.69314968 meters (distance rocket moves during this interval)

Appendix C: Let's continue now with the rest of the averaged data.

Interval number **RCL G** milliseconds	Rocket mass **RCL B** kilograms	Rocket speed **RCL C** meters/second	Distance moved **RCL D** meters	Pellet speed **RCL E** meters/second
95,000 95s	0.9075005	97.06348	485.3146	-902.9376
100,000 100s	0.9025005	102.5884	512.9391	-897.4127
105,000 105s	0.8975005	108.1440	540.7170	-891.8572
110,000 110s	0.8925005	113.7306	568.6501	-886.2705
115,000 115s	0.8875005	119.3486	596.7401	-880.6525
120,000 120s	0.8825005	124.9983	624.9888	-875.0028
125,000 125s	0.8775005	130.6802	653.3980	-869.3210
130,000 130s	0.8725005	136.3945	681.9696	-863.6067
135,000 135s	0.8675005	142.1417	710.7054	-857.8595
140,000 140s	0.8625005	147.9220	739.6073	-852.0791
145,000 145s	0.8575005	153.7360	768.6772	-846.2651
150,000 150s	0.8525005	159.5840	797.9171	-840.4172
155,000 155s	0.8475005	165.4664	827.3290	-834.5348
160,000 160s	0.8425005	171.3836	856.9150	-828.6176
165,000 165s	0.8375005	177.3360	886.6770	-822.6652
170,000 170s	0.8325005	183.3241	916.6173	-816.6771
175,000 175s	0.8275005	189.3482	946.7379	-810.6530
180,000 180s	0.8225005	195.4088	977.0411	-804.5924
185,000 185s	0.8175005	201.5064	1007.529	-798.4948
190,000 190s	0.8125005	207.6414	1038.204	-792.3598
195,000 195s	0.8075005	213.8143	1069.069	-786.1869
200,000 200s	0.8025005	220.0255	1100.125	-779.9757
205,000 205s	0.7975005	226.2756	1131.375	-773.7257
210,000 210s	0.7925005	232.5649	1162.822	-767.4363
215,000 215s	0.7875005	238.8941	1194.467	-761.1072
220,000 220s	0.7825005	245.2636	1226.315	-754.7377
225,000 225s	0.7775005	251.6739	1258.366	-748.3274
230,000 230s	0.7725005	258.1255	1290.624	-741.8758
235,000 235s	0.7675005	264.6191	1323.092	-735.3822
240,000 240s	0.7625005	271.1551	1355.772	-728.8462
245,000 245s	0.7575005	277.7341	1388.667	-722.2673
250,000 250s	0.7525005	284.3566	1421.780	-715.6447
255,000 255s	0.7475005	291.0233	1455.113	-708.9780
260,000 260s	0.7425005	297.7348	1488.671	-702.2665
265,000 265s	0.7375005	304.4916	1522.455	-695.5098
270,000 270s	0.7325005	311.2944	1556.468	-688.7070
275,000 275s	0.7275005	318.1437	1590.715	-681.8576
280,000 280s	0.7225005	325.0404	1625.198	-674.9610
285,000 285s	0.7175005	331.9849	1659.921	-668.0165
290,000 290s	0.7125005	338.9779	1694.886	-661.0235
295,000 295s	0.7075005	346.0202	1730.098	-653.9812
300,000 300s	0.7025005	353.1125	1765.559	-646.8889
305,000 305s	0.6975005	360.2554	1801.273	-639.7460
310,000 310s	0.6925005	367.4497	1837.245	-632.5517
315,000 315s	0.6875005	374.6962	1873.477	-625.3053
Interval number **RCL G** milliseconds	Rocket mass **RCL B** kilograms	Rocket speed **RCL C** meters/second	Distance moved **RCL D** meters	Pellet speed **RCL E** meters/second
Interval number	Rocket mass	Rocket speed	Distance moved	Pellet speed

RCL G milliseconds	**RCL B** kilograms	**RCL C** meters/second	**RCL D** meters	**RCL E** meters/second
320,000 320s	0.6825005	381.9955	1909.974	-618.0060
325,000 325s	0.6775005	389.3485	1946.739	-610.6530
330,000 330s	0.6725005	396.7560	1983.776	-603.2455
335,000 335s	0.6675005	404.2187	2021.090	-595.7828
340,000 340s	0.6625005	411.7376	2058.684	-588.2639
345,000 345s	0.6575005	419.3134	2096.563	-580.6881
350,000 350s	0.6525005	426.9471	2134.732	-573.0544
355,000 355s	0.6475005	434.6395	2173.193	-565.3621
360,000 360s	0.6425005	442.3915	2211.954	-557.6101
365,000 365s	0.6375005	450.2041	2251.016	-549.7975
370,000 370s	0.6325005	458.0782	2290.387	-541.9234
375,000 375s	0.6275005	466.0148	2330.070	-533.9868
380,000 380s	0.6225005	474.0148	2370.070	-525.9868
385,000 385s	0.6175005	482.0794	2410.393	-517.9222
390,000 390s	0.6125005	490.2096	2451.044	-509.7920
395,000 395s	0.6075005	498.4064	2492.028	-501.5952
400,000 400s	0.6025005	506.6710	2533.351	-493.3307
405,000 405s	0.5975005	515.0044	2575.018	-484.9973
410,000 410s	0.5925005	523.4079	2617.035	-476.5938
415,000 415s	0.5875005	531.8826	2659.408	-468.1192
420,000 420s	0.5825005	540.4297	2702.144	-459.5721
425,000 425s	0.5775005	549.0505	2745.248	-450.9513
430,000 430s	0.5725005	557.7462	2788.727	-442.2555
435,000 435s	0.5675005	566.5183	2832.587	-433.4835
440,000 440s	0.5625005	575.3679	2876.835	-424.6338
445,000 445s	0.5575005	584.2966	2921.479	-415.7052
450,000 450s	0.5525005	593.3058	2966.524	-406.6961
455,000 455s	0.5475005	602.3968	3011.979	-397.6050
460,000 460s	0.5425005	611.5712	3057.852	-388.4306
465,000 465s	0.5375005	620.8306	3104.148	-379.1712
470,000 470s	0.5325005	630.1766	3150.878	-369.8253
475,000 475s	0.5275005	639.6107	3198.049	-360.3912
480,000 480s	0.5225005	649.1346	3245.668	-350.8673
485,000 485s	0.5175005	658.7501	3293.746	-341.2518
490,000 490s	0.5125005	668.4590	3342.290	-331.5429
495,000 495s	0.5075005	678.2631	3391.311	-321.7389
500,000 500s	0.5025005	688.1643	3440.816	-311.8377
505,000 505s	0.4975005	698.1644	3490.817	-301.8376
510,000 510s	0.4925005	708.2656	3541.323	-291.7364
515,000 515s	0.4875005	718.4699	3592.344	-281.5322
520,000 520s	0.4825005	728.7793	3643.891	-271.2227
525,000 525s	0.4775005	739.1962	3695.976	-260.8059
530,000 530s	0.4725005	749.7227	3748.608	-250.2794
535,000 535s	0.4675005	760.3612	3801.801	-239.6409
540,000 540s	0.4625005	771.1141	3855.565	-228.8881
545,000 545s	0.4575005	781.9839	3909.914	-218.0183
550,000 550s	0.4525005	792.9731	3964.860	-207.0291
555,000 555s	0.4475005	804.0844	4020.417	-195.9178
<u>Interval number</u> **RCL G** milliseconds	<u>Rocket mass</u> **RCL B** kilograms	<u>Rocket speed</u> **RCL C** meters/second	<u>Distance moved</u> **RCL D** meters	<u>Pellet speed</u> **RCL E** meters/second
<u>Interval number</u>	<u>Rocket mass</u>	<u>Rocket speed</u>	<u>Distance moved</u>	<u>Pellet speed</u>

RCL G milliseconds	**RCL B** kilograms	**RCL C** meters/second	**RCL D** meters	**RCL E** meters/second
560,000 560s	0.4425005	815.3206	4076.598	-184.6816
565,000 565s	0.4375005	826.6845	4133.417	-173.3178
570,000 570s	0.4325005	838.1790	4190.889	-161.8233
575,000 575s	0.4275005	849.8072	4249.030	-150.1951
580,000 580s	0.4225005	861.5722	4307.855	-138.4302
585,000 585s	0.4175005	873.4772	4367.380	-126.5252
590,000 590s	0.4125005	885.5257	4427.622	-114.4767
595,000 595s	0.4075005	897.7211	4488.599	-102.2813
600,000 600s	0.4025005	910.0671	4550.329	-89.93537
605,000 605s	0.3975005	922.5674	4612.831	-77.43508
610,000 610s	0.3925005	935.2260	4676.124	-64.77654
615,000 615s	0.3875005	948.0469	4740.228	-51.95571
620,000 620s	0.3825005	961.0342	4805.165	-38.96837
625,000 625s	0.3775005	974.1925	4870.956	-25.81013
630,000 630s	0.3725005	987.5262	4937.625	-12.47644
635,000 635s	0.3675005	1001.040	5005.194	1.037452
640,000 640s	0.3625005	1014.739	5073.689	14.73647
645,000 645s	0.3575005	1028.629	5143.136	28.62577
650,000 650s	0.3525005	1042.714	5213.561	42.71070
655,000 655s	0.3475005	1057.000	5284.992	56.99686
660,000 660s	0.3425005	1071.493	5357.458	71.49008
665,000 665s	0.3375005	1086.199	5430.990	86.19645
670,000 670s	0.3325005	1101.125	5505.619	101.1223
675,000 675s	0.3275005	1116.277	5581.380	116.2744
680,000 680s	0.3225005	1131.663	5658.306	131.6596
685,000 685s	0.3175005	1147.288	5736.434	147.2851
690,000 690s	0.3125005	1163.162	5815.802	163.1588
695,000 695s	0.3075005	1179.292	5896.450	179.2885
700,000 700s	0.3025005	1195.686	5978.421	195.6826
705,000 705s	0.2975005	1212.353	6061.758	212.3500
710,000 710s	0.2925005	1229.303	6146.508	229.2999
715,000 715s	0.2875005	1246.546	6232.719	246.5420
720,000 720s	0.2825005	1264.090	6320.443	264.0867
725,000 725s	0.2775005	1281.948	6409.733	281.9448
730,000 730s	0.2725005	1300.131	6500.647	300.1275
735,000 735s	0.2675005	1318.651	6593.245	318.6470
740,000 740s	0.2625005	1337.520	6687.590	337.5160
745,000 745s	0.2575005	1356.752	6783.749	356.7479
750,000 750s	0.2525005	1376.361	6881.794	376.3569
755,000 755s	0.2475005	1396.362	6981.801	396.3582
760,000 760s	0.2425005	1416.772	7083.849	416.7677
765,000 765s	0.2375005	1437.607	7188.023	437.6024
770,000 770s	0.2325005	1458.885	7294.413	458.8805
775,000 775s	0.2275005	1480.626	7403.117	480.6213
780,000 780s	0.2225005	1502.850	7514.237	502.8452
785,000 785s	0.2175005	1525.579	7627.883	525.5744
790,000 790s	0.2125005	1548.837	7744.173	548.8322
795,000 795s	0.2075005	1572.649	7863.231	572.6438
Interval number **RCL G** milliseconds	Rocket mass **RCL B** kilograms	Rocket speed **RCL C** meters/second	Distance moved **RCL D** meters	Pellet speed **RCL E** meters/second
Interval number	Rocket mass	Rocket speed	Distance moved	Pellet speed

RCL G milliseconds	**RCL B** kilograms	**RCL C** meters/second	**RCL D** meters	**RCL E** meters/second
800,000 800s	0.2025005	1597.041	7985.194	597.0364
805,000 805s	0.1975005	1622.044	8110.207	622.0388
810,000 810s	0.1925005	1647.688	8238.426	647.6825
815,000 815s	0.1875005	1674.007	8370.020	674.0012
820,000 820s	0.1825005	1701.037	8505.171	701.0314
825,000 825s	0.1775005	1728.818	8644.077	728.8126
830,000 830s	0.1725005	1757.394	8786.953	757.3878
835,000 835s	0.1675005	1786.810	8934.033	786.8036
840,000 840s	0.1625005	1817.117	9085.571	817.1111
845,000 845s	0.1575005	1848.372	9241.846	848.3660
850,000 850s	0.1525005	1880.636	9403.163	880.6294
855,000 855s	0.1475005	1913.975	9569.860	913.9687
860,000 860s	0.1425005	1948.465	9742.308	948.4581
865,000 865s	0.1375005	1984.187	9920.917	984.1797
870,000 870s	0.1325005	2021.232	10,106.14	1021.225
875,000 875s	0.1275005	2059.704	10,298.50	1059.696
880,000 880s	0.1225005	2099.714	10,498.55	1099.706
885,000 885s	0.1175005	2141.393	10,706.94	1141.384
890,000 890s	0.1125005	2184.885	10,924.40	1184.876
895,000 895s	0.1075005	2230.355	11,151.75	1230.346
900,000 900s	0.1025005	2277.992	11,389.94	1277.982
Interval number **RCL G** milliseconds	Rocket mass **RCL B** kilograms	Rocket speed **RCL C** meters/second	Distance moved **RCL D** meters	Pellet speed **RCL E** meters/second

After interval 900,000 has been calculated:
669,738.5 meters = **J** = Total distance at beginning of interval
2,302.571 m/sec = **K** = Speed of rocket at beginning of interval
0.100001 kg = **L** = Mass of rocket to be accelerated during this interval
2302.581 m/sec = **N** = Speed of rocket at end of interval
1302.571 m/sec = **O** = Pellet speed after it has been accelerated
2.302576 meters = **P** = Distance moved by rocket during this interval

For each line in the table the following relationships verify internal self-consistency:
(1) If pellet speed is subtracted from rocket speed, the result will be 1000.
(2) Looking at the changes from the previous line, the difference in pellet speed will equal the difference in rocket speed, and these will equal 1/5th the difference in distance moved (within 0.0001).

There seems to be a problem with (1) above.
If the basic experiment is followed exactly, the pellet will be accelerated for 0.001 sec with a force of 1 newton. This will cause the pellet to leave the accelerator at a speed of 1000 m/sec relative to the state of motion of the center of mass of the rocket/pellet system at the beginning of the acceleration phase. The rocket will be accelerated in the opposite direction, more so as the mass of the rocket decreases. Total of both will always be greater than 1000 m/sec.

Notice in the main loop (subroutine X) that speed of the rocket after acceleration is calculated correctly, and this speed is used to produce the subdivision rocket speed average. Notice also that the pellet speed is correctly calculated based on the rocket speed at the beginning of the interval.

If the mass of the rocket is 1 kg and it is accelerated for 0.001 sec with a force of 1 newton, its resultant speed increment will be 0.001 m/sec. If the pellet is accelerated also, its resultant speed increment will be -1000 m/sec. Total speed (subtracting pellet speed from rocket speed) will be 1000.001 m/sec.

If the mass of the rocket is 0.415001 kg and we accelerate with the same parameters, the resultant speed increment will be 0.002409632748 m/sec. Total speed is then 1000.002409632748 m/sec. Since the available precision in this calculator is only 12 digits, the last digits in this total will simply disappear. The loss is 0.000000002748. The calculator simply does not have enough dynamic range to cope with the precision required in this calculation. If this error is multiplied by the number of intervals being considered (900,001), the cumulative error will be 0.0024732. This is the best accuracy we can expect.

The numeric coprocessor in the Intel Pentium processor series is able to cope with "almost" 16 decimal digits of precision, so would deliver more meaningful results in this calculation. A calculator emulation program for the HP-42s is able to operate with 24 digits of precision while running on the PC, so would have no problem at all with accumulating errors that would plague the calculators and the numeric coprocessor. See Appendix B for this program, and some sample results. The sample results are very close to the results calculated by the HP-33s. Here is HP-33s data for the remaining 100 seconds:

Interval number **RCL G** milliseconds	Rocket mass **RCL B** kilograms	Rocket speed **RCL C** meters/second	Distance moved **RCL D** meters	Pellet speed **RCL E** meters/second
905,000 905s	0.0975005	2328.013	11,640.04	1328.003
910,000 910s	0.0925005	2380.669	11,903.32	1380.658
915,000 915s	0.0875005	2436.253	12,181.24	1436.242
920,000 920s	0.0825005	2495.111	12,475.52	1495.098
925,000 925s	0.0775005	2557.651	12,788.22	1557.638
930,000 930s	0.0725005	2624.367	13,121.80	1624.354
935,000 935s	0.0675005	2695.857	13,479.25	1695.842
940,000 940s	0.0625005	2772.856	13,864.24	1772.840
945,000 945s	0.0575005	2856.286	14,281.39	1856.269
950,000 950s	0.0525005	2947.321	14,736.56	1947.302
955,000 955s	0.0475005	3047.488	15,237.39	2047.467
960,000 960s	0.0425005	3158.829	15,794.09	2158.805
965,000 965s	0.0375005	3284.157	16,420.72	2284.130
970,000 970s	0.0325005	3427.504	17,137.44	2427.473
975,000 975s	0.0275005	3594.951	17,974.66	2594.914
980,000 980s	0.0225005	3796.306	18,981.42	2796.261
985,000 985s	0.0175005	4048.977	20,244.74	3048.920
990,000 990s	0.0125005	4388.775	21,943.67	3388.694
995,000 995s	0.0075005	4912.024	24,559.77	3911.885
1000000 1000s	0.0025005	6298.218	31,486.54	5296.399

After interval 995,000 has been calculated:
968,501.0 meters = **J** = Total distance at beginning of interval
5,298.018 m/sec = **K** = Speed of rocket at beginning of interval
0.005001 kg = 5.001 grams = **L** = Mass of rocket to be accelerated during this interval
5,298.218 m/sec = **N** = Speed of rocket at end of interval
4,298.018 m/sec = **O** = Pellet speed after it has been accelerated
5.298118 meters = **P** = Distance moved by rocket during this interval

After interval 1,000,000 has been calculated:
999,978.9 meters = **J** = Total distance at beginning of interval
13,392.73 m/sec = **K** = Speed of rocket at beginning of interval
0.000001 kg = 1 mg = **L** = Mass of rocket to be accelerated during this interval
14,392.73 m/sec = **N** = Speed of rocket at end of interval
12,392.73 m/sec = **O** = Pellet speed after it has been accelerated
13.89273 meters = **P** = Distance moved by rocket during this interval